Int. J. Production Economics 149 (2014) 56–67

Contents lists available at ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

Dynamic inventory rationing with mixed backorders and lost sales

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ARTICLE INFO

Article history:
Received 14 April 2012
Accepted 4 October 2013
Available online 15 October 2013

Keywords:
Multiple demand classes
Inventory rationing
Dynamic rationing
Backorders
Lost sales

ABSTRACT

Customers may react differently when stockouts occur. In this paper we investigate the rationing policy for an inventory system with a mixture of demand classes of backorder type and lost sales type. Since the penalty cost of backorders varies with time, the priorities of demand classes also alter with time. This totally changes the problem structure compared with the classic rationing models. A dynamic rationing policy is studied in this paper by considering the dynamics of demand priorities. A Markov decision model is developed to obtain the optimal dynamic rationing levels for multiple demand classes. The results indicate that between the priority switching points, rationing levels often exhibit different patterns. For lost sales demand classes, the rationing levels always decrease as the remaining time approaches to zero. For backorder demand classes, the rationing levels increase in some parts due to declining of the priorities. The rationing levels of all demand classes finally decline to zero to reduce the inventory holding cost. The application of dynamic rationing is further extended from a single period model to a multi-period (S,T) model where unit cost has to be included. The optimal ordering policy is proved to be a myopic base stock policy and the dynamic rationing policy in the single period model can still be applied with modified time-independent penalty costs for lost sales classes. To overcome the computational complexity, a heuristic dynamic rationing policy is introduced. Due to its good outcome, implementing such a heuristic dynamic rationing policy can be a practical solution for inventory system with mixed backorders and lost sales, in order to enhance the system performance.

1. Introduction

In the traditional inventory models, demand is considered to be homogeneous and often fulfilled with the first-come-first-server principle. Nowadays customers are often classified into different groups, for examples VIPs, premium and ordinary ones, who are considered to bring different values to companies. In a finite time horizon without production and replenishment opportunity, inventory is generally limited. Thus at certain time points, inventory is preferred to be reserved for the demand classes with high added value in order to maximise profits or avoid expensive penalty costs. Inventory rationing is one method to achieve these goals.

Before implementing a rationing policy, we often need to define the priorities of demand classes. Most literature assigns priorities to demand classes according to their penalty costs which often have the same structure. This implies that customers are of the same type, i.e. they have similar behaviour in facing stockouts but the values of penalty cost vary. In practice customers may exhibit different purchasing behaviours, in particular with different reactions to stockouts. Some are willing to wait until demand is fulfilled, and the others walk away immediately or purchase from other sources. These two responses lead to backorders and lost sales, respectively. There are plenty of industrial cases with such kinds of mixed phenomenon. For example, e-commerce companies usually perform as suppliers for large business customers as well as serve individual consumers. When the inventory is out of stock, the individual consumers will likely switch their purchasing to other websites, while the large business customers will wait because of price discounts or constraints in the contracts.

When we apply the rationing polices to different classes which react differently to stockouts, the decisions become more complicated. If there is a long remaining time till the end of period (or next replenishment), we may reject the customer demand of lost sales type in order to reserve inventory to the ones of backorder type which may have a high penalty cost due to a long waiting time. On the other hand, when the decision point is approaching the next replenishment, we may ask the customers of backorder type to wait for a short while and use the inventory to avoid further lost sales, because the penalty cost associated with backorders becomes less than that of lost sales. In summary, due to different dimensions of the penalty costs, we need to switch the protection focus and consequently adjust the amount of reserved inventory at different decision points along with time.

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http://dx.doi.org/10.1016/j.ijpe.2013.10.004

In this paper, we therefore study the rationing policies in an inventory system with a mixture of backorders and lost sales. Since at least one demand class (backorder type) has a penalty cost changing over time, the importance of demand classes should also change along with time and consequently the demand priority may switch. In this circumstance, a dynamic rationing policy should be employed. Thus, in this paper we first develop a model for dynamic rationing in a single period. Such a system is studied because the pattern and advantages of dynamic rationing can be explored. We then apply dynamic rationing in a multi-period system and check its impact on ordering policy and cost. For the purpose of making the dynamic rationing policy easy to be implemented, we also introduce a simple algorithm with a heuristic policy for obtaining the rationing levels.

The structure of this paper is as follows. In Section 2 we introduce two streams of literature related to our study. In Section 3, we develop a discrete Markov model for obtaining the optimal dynamic rationing levels in a single period. The dynamic rationing is then extended to a multi-period system in Section 4. Then a heuristic dynamic rationing policy is developed in Section 5. The conclusions are drawn in Section 6.

2. Literature review

Current research is related to two streams of the literature. One is the research on inventory models with backorders and lost sales, but without the consideration of demand priorities. The other is the research on dynamic rationing policies with multiple demand classes.

The mixture of backorders and lost sales has been considered in the research on inventory control for many years. But demand of both types is often treated equally in these studies, i.e. it is satisfied on the first-come-first-serve basis without distinguishing the priorities of demand classes. Montgomery et al. (1973) first study such a mixed system with backorders and lost sales assuming deterministic and stochastic demand, respectively. In order to facilitate the calculation of near-optimal solution, the stockout time is ignored in the same way as the lost sales model in Hadley and Whitin (1963). Rosenberg (1979) reformulates Montgomery's model with deterministic demand so that the optimal solution can be easily obtained. Kim and Park (1985) consider a (r,Q) policy with a mixture of lost sales and time-dependent backorders. They consider the time duration of stockout (which is ignored in Montgomery et al.,1973) and obtain the exact optimal solution of the model. Rabinowitz et al. (1995) study another kind of mixed inventory model. Initially, all demand is backlogged when it runs out of stock. However the system only allows a maximum number of backorders in a cycle so that the rest of stockouts has to be lost. Ouyang et al. (1996) extend this problem with a variable lead time which increases uncertainties. Again we note that in all the studies mentioned above, demand is homogeneous, i.e. it is treated in the same manner.

The second stream of research is on inventory rationing. It assigns different priorities to different demand classes. A critical rationing level is often assigned. When the inventory declines to this critical level, it stops serving the low priority demand class. Traditionally, the critical level is set to be static for the sake of simple implementation. However, such a static rationing policy also misses the chance of further improving system performance. The rationing decision (level) depends on the remaining time to next replenishment, priorities of demand classes and current inventory status. Since these factors are changing with time, a dynamic rationing decision (level) should reflect better the continuously updated information. Melchiors (2003) studies the dynamic rationing for two lost sales demand classes. He formulates the problem by using a Markov decision model. The time is divided into several intervals, each of which has a static rationing level. As the number of interval increases, the rationing levels become dynamic, while the computation time increases as well. We also previously addressed a problem with two classes of lost sale type in Wang and Tang (2012). We find that a combination of a static rationing policy before lead time and a dynamic rationing during lead time can outperform other rationing policies. However, the dynamic rationing policy in above studies is limited to two lost sales classes. Teunter and Haneveld (2008) study the dynamic rationing with two backorder demand classes in a single period model. They do not discretise the time, but analyse the marginal cost to determine the optimal remaining time for each rationing level. The rationing levels can be calculated backwards at any time point starting from the end of period. Chew et al. (2013) also investigate the dynamic rationing problem with two backorder classes. After discretising the time into small intervals, they formulate the problem as a Markov decision model to obtain the optimal rationing level. A base stock level is obtained for both zero lead time case and non-zero lead time case. Hung et al. (2012) further extend the research with general demand processes. Fadiloglu and Bulut (2010) examine the dynamic rationing policy with backorders or lost sales. Instead of defining directly the dynamic rationing levels, they combine the on-hand inventory level and outstanding replenishment orders (the latter is represented by an exponential function). With this transformation, they develop dynamic rationing level by adopting the same approach as in static rationing. The rationing problem in a make-to-stock production system is studied in Ha (1997a, 1997b). He assumes exponential production times and shows that the optimal rationing levels are monotone or stationary. The similar analysis is extended to Erlang distributed production time in Ha (2000). Note that all the studies mentioned above deal with the same type of stockouts in the dynamic rationing policy.

There have been few studies on inventory rationing with a mixture of backorders and lost sales. Benjaafar et al. (2010) study the optimal control of a production inventory system with two customer classes. The admission decision includes whether to backorder a demand or to reject (lost sale) a demand. Also with one demand class, Bhaskaran et al. (2010) study the optimal admission decision to determine whether to fulfil backorder or refuse when demand occurs. In fact these papers do not consider the different responses from customer behaviour perspective, but rather assume the supplier can define or determine the consequence of stockouts for treating excess demand. Nevertheless, these two studies again illustrate the common existence of mixed types of stockouts in practice.

In the current study, we consider a dynamic rationing policy for multiple demand classes with a mixture of lost sales and backorders. This dynamic rationing is further implemented to two multi-period inventory systems. Such a study fills the research gap in the literature. Exploring the pattern of dynamic rationing and its associated benefit in such a complicated environment should enhance our knowledge in inventory rationing. The study results should also guide the managerial principle for inventory rationing in practice.

3. Dynamic rationing in a single period

We first study a single period inventory system in order to understand the pattern of dynamic rationing with a mixture of backorders and lost sales. The system satisfies demand from J demand classes indexed by i, $i \in \{1,\dots,J\}$. The demand of each class is Poisson distributed with rate λ_i. Here we introduce a general