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Flexible service policies for a Markov inventory system with two demand classes

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ABSTRACT

This paper explores flexible service policies for an (r, Q) Markov inventory system with two classes of customers, ordinary and prioritized customers. When the on-hand inventory drops to pre-determined safety level r, arrival ordinary customers receive service at probability p. Firstly, the inventory level state transitions equation is set up. The steady-state probability distribution and the system's performance measures which are used for the inventory control are derived. Next, a long-run average inventory cost function is established and a mixed integer optimization model is set up. And, an improved genetic algorithm for the optimum control policies is presented. Finally, the optimal inventory control polices and the sensitivities are investigated through numerical experiments.

1. Introduction

Inventory systems usually satisfy demands from more than one types of customer, each of which may possess respective characteristics, such as affordable price and quality of services. Typically, the variable demands of different customers result in various service priority. The type of priority inventory demand is classically categorized into booked orders and unscheduled orders. Booked orders, which are from long-term contracts and have much higher shortage lost, must be satisfied preferentially, whereas unscheduled orders, which are from the stochastic demand, may bear lower shortage cost and can be lost. The real-life situations and extensive implications drive us to consider the inventory system with two demand classes.

The practices of inventory management face multiple classes of demands. Veinott (1965) considered a critical level policy for solving the problem of several demand classes in inventory systems. Nahmias and Demmy (1981), Dekker and Kleijn (1998) and Deshpande et al. (2003) also studied inventory control problems with different classes of customer. Hung et al. (2012) consider the dynamic rationing problem for inventory systems with multiple demand classes and general demand processes. They assume that back orders are allowed. The aim is to find the threshold values for this dynamic rationing policy. In the lost sales case, an important issue in the inventory systems is the inventory control policies of optimal inventory (Ha, 1997; Dekker et al., 2002). Melchiors et al. (2000) derived a continuous review inventory system with lost sales and two demand classes. They proposed a formula for the total expected cost and presented a numerical procedure for optimization. But, they could not prove convexity of the cost function. Sapna Isotupa (2006) analyzed a similar model using exponentially distributed lead-times, and then established a long-run expected cost function. He proved that cost function is pseudo-convex in both parameters s/r and Q. Other scholars such as Berman and Kim (1999), Berman and Sapna (2001) and Schwarz et al. (2006) examined queueing-inventory systems over the last two decades. They investigated the behavior of service systems with an attached inventory. They defined a Markovian system process and then used classical optimization methods to find the optimal control policy of the inventory (Krishnamoorthy et al., 2006; Manuel et al., 2008). Recently, Zhao and Lin, 2011 investigated a queueing-inventory system with two classes of customers and found a priority service rule to minimize the long-run expected waiting cost. Ioannidis (2011) propose a simple threshold type policy for a two-class system in which the production, service, and back-ordering decisions are integrated. He proposed a simple threshold type heuristic policy for the joint control of inventories and backorders.

Even though there are some models in the literature that incorporate two or multiple classes of demands considering possible lost sales for rejecting ordinary customers' demands, there is a lack of studies using more flexible service polices. This paper presents flexible service policies for an (r, Q) Markov inventory system with two classes of customers. Three major differences from the literature are outlined here. Firstly, our paper introduces a priority parameter p, which is different from the

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previous papers by Sapna Isotupa (2006) and Zhao and Lin (2011) on Markov inventory systems with two classes of customers. The parameter $p(0 \le p \le 1)$ is used for controlling the application of priority. When ordinary customers arrive, the system makes a decision whether or not to offer service; when prioritized customers arrive, they are served in priority. If $p=1$, there is no priority service in the inventory system. If $p=0$, there is a strict priority service in the inventory system. In this case, our model is the same as Sapna Isotupa (2006). If $0<p<1$, when the on-hand inventory drops to the safety level r, arrival ordinary customers will receive service at probability p. Secondly, our paper establishes a mixed integer optimization model with integer variables (r, Q) and real variable (p). We adopt a real coded genetic algorithm genetic named MI-LXPM for solving integer and mixed integer constrained optimization problems. Lastly, we conduct eight numerical experiments for investigating the sensitivities of system parameters and reveal more management insights than the literature.

We will describe our Markov inventory model in Section 2 and derive the steady-state performance measures in Section 3. In Section 4, we will first establish a long-run average inventory cost function and then prove that the cost function is pseudo-convex about r and Q for fixed parameter p. A mixed integer optimization model will be established and the MI-LXPM algorithm will be presented in Section 5. This will be followed by some numerical experiments that investigate the service discipline with different cost in Section 6. The paper will be concluded in Section 7.

2. The model description

We consider a single server Markov inventory system based on the following assumptions: (1) the system serves two classes of customers —prioritized customers and ordinary customers; (2) the arrival process for both classes is state independent and each customer needs exactly one item from the inventory; (3) prioritized customers arrive according to Poisson process with intensity λ_1 and ordinary customers arrive according to Poisson process intensity λ_2; (4) the service discipline is first-come-first-service (FCFS) and the service time is 0 (compared to the lead-time for the order, the customers' order processing time can be omitted). The lead-time is exponentially distributed with parameter μ; and (5) the replenishment is never interrupted and there is at most one outstanding order at any time.

An (r, Q) policy is the most popular ordering policy in relation to inventory control. It was well discussed in queueing-inventory models in Schwarz et al. (2006), Zhao and Lin (2011), etc. Some scholars use (s, Q) policy as (r, Q) policy (Melchiors et al., 2000; Sapna Isotupa, 2006). However, as (r, Q) is more popularly used, we apply the continuous review (r, Q) policy instead of (s, Q) with an additional flexible service. As and when the on-hand inventory drops to a fixed level r(safety inventory level), an order for fixed $Q(>r)$ units is placed. The condition $Q>r$ ensures that there is no perpetual shortage. Hence the maximum on-hand inventory is $r+Q$. When the on-hand inventory is more than the safety level r, the two classes of customers who have arrived can both be served. However, when the on-hand inventory drops to the safety level r, arrival ordinary customers receive service at the probability p. Those ordinary customers who do not receive service are lost. When the inventory is empty, both classes of customers get lost.

3. The steady-state performance measures

Let $I(t), t \ge 0$ be the on-hand inventory level at time t. From the model assumptions, the state space of $I(t)$ is $E=\{0,1,\cdots,r,\cdots,Q,1+Q,\cdots,r+Q\}$. For the Poisson input process and the exponential distribution lead-time, the inventory level state next period does not depend on any past states but on the current state. The inventory level process $I(t)$ constitutes a Markov process on state space E.

We denote by $P(i,j,t)$ the state-transition probability from the state i at time 0 to state j at time t,$P(i,j,t)=P\{I(t)=j|I(0)=i\}$, $i,j \in E$. We define the steady-state probability distributions for $I(t)$ as $P(j)=\lim_{t\to\infty}P(i,j,t), j\in E$.

In the long run equilibrium, the steady-state probability distributions of the inventory level $P(j)$ satisfy Eqs. (1)–(6). The balance equations can be formulated by the fact that transition out of a state is equal to transition into a state for a Markov process. For example, if the inventory level state j lines in the range $Q \le j \le Q+r-1$, the equation is presented in Eq. (2). When j is within this range, there is no order pending, and then transition beyond this state can be only due to either an ordinary demand arrival or a prioritized demand arrival, which is presented on the left-hand side of Eq. (2). Either an ordinary demand or a priority demand in state $j+1$ will reduce the inventory level by one unit, thus bring it to state j. State j can also be reached from state $j-Q$ when a replacement arrives. The only two possible ways of reaching state j are reflected on the right-hand side of Eq. (2).

$$(\lambda_1+\lambda_2)P(Q+r)=\mu P(r), \tag{1}$$

$$(\lambda_1+\lambda_2)P(j)=(\lambda_1+\lambda_2)P(j+1)+\mu P(j-Q), j=Q, \quad Q+1,\cdots,Q+r-1, \tag{2}$$

$$(\lambda_1+\lambda_2)P(j)=(\lambda_1+\lambda_2)P(j+1), \quad j=r+1,r+2,\cdots,Q-1, \tag{3}$$

$$(p\lambda_2+\lambda_1+\mu)P(r)=(\lambda_1+\lambda_2)P(r+1), \tag{4}$$

$$(p\lambda_2+\lambda_1+\mu)P(j)=(\lambda_1+p\lambda_2)P(j+1), \quad j=1,2\cdots,r-1, \tag{5}$$

$$\mu P(0)=(\lambda_1+p\lambda_2)P(1). \tag{6}$$

The above set of equations together with the normalizing condition written as $\sum_{j=0}^{Q+r}P(j)=1$ determine the steady-state probability distributions uniquely. We solve Eqs. (1)–(6) by means of recursive process, and get

$$P(j)=\left(1+\frac{\mu}{\lambda_1+p\lambda_2}\right)^{j-1}\frac{\mu}{\lambda_1+p\lambda_2}P(0), \quad j=1,2,\cdots,r, \tag{7}$$

$$P(j)=\left(1+\frac{\mu}{\lambda_1+p\lambda_2}\right)^{r}\frac{\mu}{\lambda_1+\lambda_2}P(0), \quad j=r+1,r+2,\cdots,Q, \tag{8}$$

$$P(j)=\left[\left(1+\frac{\mu}{\lambda_1+p\lambda_2}\right)^{r}-\left(1+\frac{\mu}{\lambda_1+p\lambda_2}\right)^{j-Q-1}\right]\frac{\mu}{\lambda_1+\lambda_2}P(0),$$
$$j=Q+1,Q+2,\cdots,Q+r, \tag{9}$$

$$P(0)=\frac{\lambda_1+\lambda_2}{\lambda_1+p\lambda_2+[Q\mu+(1-p)\lambda_2]\left(1+\frac{\mu}{\lambda_1+p\lambda_2}\right)^{r}}. \tag{10}$$

Inserting (10) in (7)–(9) respectively, we have the analytical steady-state probability distributions of the inventory level.

Let $\bar{I}$ denote the average inventory level. Using $\bar{I}=\sum_{j=1}^{r+Q}jp(j)$, we have

$$\bar{I}=\left(1+\frac{\mu}{\lambda_1+p\lambda_2}\right)^{r}\left[\frac{r(1-p)\lambda_2}{\lambda_1+\lambda_2}+\mu\frac{Q^2+2Qr+Q}{2(\lambda_1+\lambda_2)}-Q\frac{\lambda_1+p\lambda_2}{\lambda_1+\lambda_2}-\frac{(1-p)\lambda_2}{\mu}\frac{\lambda_1+p\lambda_2}{\lambda_1+\lambda_2}\right]P(0)$$
$$+\left(Q+\frac{(1-p)\lambda_2}{\mu}\right)\left(\frac{\lambda_1+p\lambda_2}{\lambda_1+\lambda_2}\right)P(0). \tag{11}$$

Let us denote

$\bar{R} \equiv$ the mean reorder rate
$\bar{\psi}_1 \equiv$ the mean shortage rates for the prioritized customers
$\bar{\psi}_2 \equiv$ the mean shortage rates for the ordinary customers