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The American put under transactions costs

Stylianos Perrakis[a,*], Jean Lefoll[b]

[a]Department of finance, The John Molson School of business, Concordia University, Montréal, Québec, Canada, H3G 1M8
[b]HEC and International Finance Laboratory, University of Geneva, 1211 Genève 4, Switzerland

Abstract

This paper examines the optimal super-replication of American put options with physical delivery of the underlying asset, such as stock options, by means of a stock-plus-riskless asset portfolio. The framework of the analysis is the binomial model with proportional transactions costs on stock transactions. The paper extends the model for European options, originally presented in Merton (Geneva Papers Risk Insurance 14 (1989) 225) and Boyle and Vorst (J. of Finance 47 (1992) 271), and generalized in Bensaid et al. (Math. Finance 2 (1992) 63). The optimizing framework of this latter study is adapted to put options held by investors and perfectly hedged by a market maker, and to put options written by investors and both held and hedged perfectly by a market maker. It is shown that a unique optimal super-replicating portfolio exists at every node of the binomial tree for the long option, as well as for the short option when transactions costs are low.

JEL classification: G13

Keywords: Pricing; Transactions costs; American options; Put

1. Introduction

This paper examines the pricing of American put options by a *perfectly-hedged* market maker when there are transactions costs to be paid on the underlying stock. Thus, it extends European stock option pricing under perfect hedging, transactions costs, and binomial stock returns, formulated by Merton (1989), and extended by Boyle and Vorst (1992) BV. A similar extension, to American options on dividend-paying stocks under transaction costs, was done in an earlier study (Perrakis and Lefoll, 2000).

* Corresponding author.
E-mail address: sperrakis@mercato.concordia.ca (S. Perrakis).

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While the perfect-hedging assumption may appear extreme for organized option markets, it does offer a useful benchmark case for the derivation of option bounds, within which the option bid/ask prices must lie. Further, this assumption makes our results also useful to situations where there is a need for option replication such as, for instance, in portfolio insurance. Last, it allows the creation of options in cases in which no organized options market exists, as in most emergent financial markets. Thus, while most results are expressed in terms of option pricing by a perfectly-hedged market maker, they also extend to these other important cases.

In the option pricing models of the classic studies of Black and Scholes (1973) and Merton (1973), the call option is perfectly and continuously replicated by a stock-plus-riskless-asset portfolio. The introduction of fixed transactions costs every time this portfolio is being rebalanced makes such a policy infeasible in a continuous time model. For this reason a number of papers have tackled the problem of portfolio selection and/or option pricing under transactions costs, both in continuous time [1] and on a binomial lattice.

Since perfect option replication is infeasible in the continuous time models, those studies that dealt with option pricing specified either approximate replication at predetermined and exogenously given times, or expected utility-based portfolio selection under transactions costs. By contrast, the Merton–BV approach replicates both long and short call options at every node of the binomial lattice. While the replication of the long option is feasible in all cases, the replication of the short option requires some restrictions on parameter values. These restrictions are satisfied when transactions costs are 'small' for the chosen number of lattice steps, in a sense that will become more precise in Section 4 of this paper.

Merton solved the replication problem when the option has only two periods to expiration; BV extended the Merton model to any number of periods. An important study by Bensaïd et al. (1992) BLPS, derived an algorithm to compute optimal perfect-hedging policies for an intermediary that issues long or short options (which they named *super-replication*), for several types of European options under binomial returns without necessarily replicating the option at every node. [2] The BLPS study found contrasting results for the important cases of physical delivery and cash settlement options: while the intermediary finds it optimal to replicate everywhere physical delivery long options, such a policy is suboptimal for cash settlement options, unless transactions costs are 'small', in the same sense as in the Merton–BV studies.

In spite of its generality and powerful theoretical insights, the BLPS algorithm is rather difficult to apply as stated for a large number of periods to expiration, since

[1] See Leland (1985), Constantinides (1986), Hodges and Neuberger (1989) and Dumas and Luciano (1991).

[2] Edirisinghe et al. (1993) also presented a two-stage dynamic programming algorithm for minimum cost hedging of an option without necessarily replicating it; see also Boyle and Tan (1994) for more on their method. The two-stage algorithm, however, was developed for options whose method of settlement is up to the seller, and it is not clear how it could be extended to cover the more realistic physical delivery and cash-settlement options. Further, for options with settlement up to the seller BLPS (1992) presented a closed form solution in their Theorem 4.