Int. J. Production Economics 137 (2012) 11–18

Contents lists available at SciVerse ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

Economic order quantity and purchasing price for items with imperfect quality when inspection shifts from buyer to supplier

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ARTICLE INFO

Article history:
Received 24 January 2011
Received in revised form
7 November 2011
Accepted 11 January 2012
Available online 20 January 2012

Keywords:
Economic Order Quantity (EOQ)
Imperfect quality
Inspection
Purchasing
Buyer-supplier relationship

ABSTRACT

Traditional Economic Order Quantity (EOQ) models, implicitly assume that all items that are received are perfect. Although recent EOQ models for items with imperfect quality, which relax this assumption, are closer to real-world problems, they implicitly assume that suppliers do not conduct a full inspection. In this paper, we study the relationship between buyer and supplier with regard to conducting the inspection and resulting in a change the buyer's economic order quantity and purchasing price. We model and analyze the problem under two conditions: (1) assuming there is no relationship between the buyer's selling price, buyer's purchasing price, and customer demand; (2) assuming there is relationship between the buyer's selling price, buyer's purchasing price, and customer demand. Numerical examples are provided to illustrate the models.

1. Introduction

Traditional economic order quantity (EOQ) models offer a mathematical approach to determine the optimal number of items a buyer should order to a supplier each time. One major implicit assumption of these models is that all the items are of perfect quality. However, this is not always the case, as in some situations a percentage of the items is imperfect. Porteus (1986) and Rosenblatt and Lee (1986) were among the first to study the effect of imperfect items on EOQ and EPQ (economic production quantity) models. Following them, several studies have incorporated the effect of imperfect items into different EOQ and EPQ models, most notably Salameh and Jaber (2000), who considered a situation in which an average *p*% of all items ordered are imperfect. The buyer conducts an inspection of all the items to separate the imperfect items from the perfect ones, after which the imperfect items are assumed to be sold as a single batch at the end of inspection process. Formulating the problem, the optimal order quantity is derived. Salameh and Jaber's (2000) model has been extended in several directions.

For example Goyal and Cárdenas-Barrón (2002) corrected a small error in the original model. Rezaei (2005), Papachristos and Konstantaras (2006), Wee et al. (2007) and Chang and Ho (2010) studied the problem by considering the occurrence of shortage. Chung and Huang (2006) identified the optimal order quantity with imperfect items for retailers when delay in payment is allowed. Hsu and Yu (2009) formulated the problem to determine the optimal order quantity under a one-off discount. Chan et al. (2003) divided imperfect items into three categories: imperfect items that can be sold at a lower price, imperfect items that can be reworked and imperfect items that should be rejected, after which they devised a mathematical model to determine the EPQ. Wang (2005) proposed a mathematical model to determine the EPQ and optimal inspection. Haji and Haji (2010) formulated and solved the problem in situations where imperfect items are reworked with a random rate. In a recent study, Maddah and Jaber (2008) corrected some of the flaws of the original study by Salameh and Jaber (2000), which, although mathematically interesting, led to no significant changes in the final results.

Some researchers considered imperfect items in the context of buyer-supplier relationships. Huang (2002, 2004) and Goyal et al. (2003), for example, formulated models to determine the optimal integrated buyer-supplier inventory policy for imperfect items and found that joint decision-making can reduce the expected annual cost of inventory significantly. Chen and Kang (2007, 2010) formulated the problem after considering the delay in payment. They assumed that the supplier can increase the warranty cost to maintain the long-term relationship. Rezaei and Davoodi (2008) formulated a model to determine the optimal

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doi:10.1016/j.ijpe.2012.01.005

lot-size including imperfect items and select the suppliers simultaneously, while Lin (2009) formulated a model for a single-supplier/single-buyer relationship to determine the optimal lot-size when some items are imperfect. Quality of received items has been also recognized in the literature of supplier selection as one of the most important criteria (see for example Rezaei and Davoodi, 2008, 2011). For a detailed review and discussion of the extensions of EOQ model for imperfect quality items, see Khan et al. (2011).

A review of existing literature reveals that several extensions of the problem have been proposed. However, in all the relevant studies, the inspection process is assumed to be conducted by the buyer. In this paper, we study the problem from a different perspective and propose a suitable framework to determine how the buyer can help the supplier carry out the inspection and reduce the imperfect rate. The rest of this paper is organized as follows. Section 2 contains the mathematical modeling of the problem under two different conditions. In Section 3, numerical examples are presented and Section 4, finally, contains the conclusion and suggestions for future research.

2. Mathematical modeling

2.1. Notations

The following notations are used in this paper.

K	Fixed ordering cost
c	Regular purchasing price per unit (if imperfect rate $p>0$)
Mc	Maximum purchasing price per unit (if imperfect rate $p=0$)
s	Selling price per perfect unit
h	Holding cost per unit per time unit
x	Inspection rate per time unit
p	Imperfect rate
$E[p]$	Expected imperfect rate
d	Screening (inspection) cost per unit
D	Demand rate per time unit
v	selling price per imperfect unit
T	Ordering cycle duration
y	Order quantity

2.2. Problem definition and research questions

In contrast to traditional EOQ models, which implicitly assume that all items are completely perfect, Salameh and Jaber (2000) have formulated the problem in situations where not all items are perfect. The imperfect items are separated from perfect ones by a full inspection and are used in another inventory situation. As the authors indicate, the electronic industry is a good example of such a situation.

The implicit assumption of Salameh and Jaber's (2000) model, however, is that the supplier does not perform a full inspection; otherwise the received batches are expected to be completely perfect. In fact, the very presence of imperfect items in a batch depends on whether or not the supplier carries out a full inspection, which is why we outline two different possible scenarios here:

Scenario 1. The supplier does not perform a full inspection and, as a result, the batches received by the buyer contain some imperfect items. This implies that the buyer should conduct a full inspection.

Scenario 2. The supplier performs a full inspection and, as a result, the batches received by the buyer contain no imperfect items.

The first scenario was formulated and analyzed in Salameh and Jaber (2000), while the second scenario is the implicit assumption of traditional EOQ models.

Based on these two scenarios, we examine the following research questions:

1. If the buyer is able to select between two suppliers, one of whom fits the first scenario, while the other one fits the second scenario (performs a 100% inspection), which one is preferred by the buyer?
2. If the supplier belongs to the former category (does not perform a 100% inspection), does it make sense for the buyer to be of assistance the supplier (by paying more for each item) to improve the supplier's production quality or to help the supplier perform a full inspection?

At face value, it appears that the answer to the first question is simply to select a supplier belonging to the second scenario. However, if the conditions are the same, it is to be expected that, in the second scenario, the unit price of items offered by the supplier are higher than those in the first scenario, which means there is trade-off between quality and price. To answer this question, we need to identify the maximum purchasing price (Mc) the buyer is willing to pay to a supplier belonging to the second scenario. If the unit price is less than Mc, the second scenario is preferred, if not the first scenario is preferred. However, the comparison problem may be more complicated when a supplier belonging to the first scenario also agrees to conduct a full inspection (see Section 2.5 for more details).

To answer the second question, knowing Mc, we can also determine how much more the buyer is willing to pay for a full inspection by the supplier. To improve production quality and reduce the percentage of imperfect items, the buyer can also pay more for each item, which is discussed later in this section (here it is still the buyer who conducts the full inspection). Note that the additional payment to improve the supplier's production quality is different from the additional payment designed to help the supplier carry out a full inspection.

As we can see, the purchasing price changes when a supplier belonging to the second scenario in the first research question is selected or when the goal is to improve the supplier's production quality. Changes in buyer's purchasing price usually influence the buyer's selling price which in turn, assuming the relationship between the buyer's selling price and customer demand, affects the customer demand and economic order quantity.

We model and solve the problem under two different conditions. First, (in Section 2.3) we assume that buyer's selling price does not depend on the purchasing price, and second, (in Section 2.4), in a situation that is closer to real-world problems, we assume that the purchasing price influences the buyer's selling price and consequently customer demand.

2.3. Determining Mc assuming there is no relationship between the buyer's selling price, the purchasing price, and customer demand

According to Salameh and Jaber (2000) and Maddah and Jaber (2008), the buyer's expected profit per ordering cycle is as follows.

$$\begin{aligned} TP(y) = {} & \text{total sales of good quality items} \\ & + \text{total sales of imperfect quality items} \\ & - \text{ordering cost} - \text{purchasing cost} - \text{inspection cost} \\ & - \text{holding costs} \end{aligned}$$