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Doubling an investment

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Abstract

We study the issue of optimal long-term portfolio management in continuous time multi-asset financial markets. Rather than following the abstract notion of 'utility' and its implied paradigm of 'maximization of expected utility' we suggest a different approach: The investor sets a *goal*—such as reaching a desired fortune level, or doubling the initial investment—and then operates to *minimize* the *expected time-to-goal*, i.e., achieving the goal as quick as possible.

We assume the 'standard model' of multi-asset financial markets where assets are governed by correlated Geometric Brownian motion dynamics, and study optimality under the criteria of 'minimization of the expected time-to-goal'. We *explicitly* compute: (i) the optimal holding strategies; (ii) the dynamics and behavior of the optimal investment portfolios; and, (iii) the statistics—mean, variance, and Laplace transform—of the time-to-goal (under the optimal investment strategy).

Also, an investment paradox arising in this context—in which some portfolios have exponential mean growth but have a positive probability of *never* doubling their initial value—is discussed and explained.

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1. Introduction

Consider an investor operating in a multi-asset financial market $\mathscr{M}$. Let H denote the investor's (dynamic) holding strategy and $P_H = (P_H(t))_{t \geqslant 0}$ denote the value process of the investor's portfolio under the strategy H. According to the von Neumann–Morgenstern theory of expected utility [1], the investor's preferences are quantified by a *utility function* (the von Neumann–Morgenstern utility function), and the investor

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operates so as to *maximize* his *expected utility*. Hence, having set a time horizon T, the investor arrives at the optimization problem

$$\sup_H \mathbf{E}[U(P_H(T))] , \tag{1}$$

where U is the investor's utility function, and where H runs over all admissible holding strategies.

Problems of this type, as well as various problems regarding 'optimal consumption', were studied extensively in the literature—we refer the reader to Refs. [2–4].

Since its inception, the von Neumann–Morgenstern theory of expected utility played a dominant role in the financial–economic literature (see Ref. [5] and references therein). The 'paradigm of expected utility' has, however, serious drawbacks. On the one hand, the notion of utility is rather abstract and difficult to implement practically in real-life situations—indeed, most investors are incapable of even defining their utility functions. On the other hand, the axiomatic framework underlying the theory of expected utility is questionable, due to systematic violations of the theory's hypotheses which were repeatedly observed in experimental studies (see, for example, Refs. [6–10]).

In this work we introduce and study an alternative approach, suitable for long-term investors, which comes around the inherent drawbacks of 'utility'. Rather than setting a time horizon T and operating so as to maximize the investor's expected utility at time T, we suggest that the investor set himself a goal—such as reaching a desired fortune level, or doubling the initial investment—and operate so as to *minimize* the *expected time-to-goal*. Hence, if τ_H denotes the time-to-goal under the holding strategy H, the investor would arrive at the optimization problem

$$\inf_H \mathbf{E}[\tau_H] \tag{2}$$

(again, H running over all admissible holding strategies).

In other words, the algorithmic approach

(i) Set a time horizon T,
(ii) Maximize the expected utility at time T

is replaced by

(i) Set a goal G,
(ii) Minimize the expected time-to-goal G.

The first approach is based on the abstract notion of 'utility', whereas the second is nothing but the very straightforward and intuitive concept of achieving one's goal "as quick as possible!".

An analogous approach of 'goal-based' portfolio management was recently introduced by Cohen and Natoli [11], exploring the case of an investor seeking to optimize his portfolio's performance in a finite time horizon setting, subject to preset minimum-acceptable yield and utility goal-levels.

The paper is organized as follows: In Sections 2 and 3, we review multi-asset financial markets: general setting, portfolio management, and the 'standard model' of