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Scheduling problems with multiple maintenance activities and non-preemptive jobs on two identical parallel machines ☆

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ABSTRACT

This paper deals with the problem of processing a set of n jobs on two identical parallel machines. In order to reduce the probability of machine breakdown with minor sacrifices in production time, the machines cannot process the jobs consecutively, they need to be maintained regularly (here we assume that the largest consecutive working time for each machine cannot exceed an upper limit T). Two scheduling models are considered. In the first model, the maintenance activities are performed periodically and the objective is to schedule the jobs on two machines such that the makespan is minimized. In the second model, the maintenance activities are determined jointly with the scheduling of jobs, and the objective is to minimize the total completion time of jobs. For the first problem, we introduce an $O(n^2)$ time algorithm named MH_{FFD} and show that the performance ratio of MH_{FFD} is at most $\max\{1.6+1.2\sigma, 2\}$, where $\sigma \triangleq t/T$, t is the amount of time to perform each maintenance activity. For the second problem, we apply the classical SPT algorithm to it and show that the worst-case bound of SPT algorithm is no more than $1+2\sigma$. We also point out that for the case of single machine, if the SPT schedule has three batches, then the upper bound of SPT algorithm can be reduced from the known result 21/17 to 11/9 under the assumption that $t<T$.

1. Introduction

The majority of machine scheduling literature assumes that the machines are available for processing jobs at all times during the planning horizon. However, this assumption may not be valid in a real production situation due to preventive maintenance (a deterministic event) or breakdown of machines (a stochastic phenomenon). Uncertain breakdowns will make the shop behavior hard to predict, thus reducing the efficiency of the production system. Maintenance activity can reduce the breakdown rate with minor sacrifices in production time. The role and importance of industrial maintenance has increasingly been recognized by decision makers (Pinjala et al., 2006; Alsyouf, 2007, 2009; Dowlatshahi, 2008). Therefore, scheduling the maintenance in manufacturing systems has gradually become a common practice in many companies. However, according to practical experience, it sometimes can be found that some of the machines are awaiting maintenance while there are jobs waiting to be processed by these machines. This is due to the lack of coordination between maintenance planning and production scheduling. Therefore, there is a need to develop efficient scheduling methods to improve the situation by deriving a satisfactory schedule that considers both jobs and maintenance activities simultaneously. With proper planning of the maintenance activities, the shop can improve production efficiency and safety, resulting in increased productivity and heightened safety awareness (Art et al., 1998).

The scheduling of maintenance activities can be determined either before the scheduling of jobs, or jointly with the scheduling of jobs. In the first case, the maintenance periods are known and fixed in advance. Thus the problem of scheduling jobs with this kind of maintenance reduces to the problem often referred in the literature as *scheduling with machine availability constraints*. Adiri et al. (1989), Lee and Liman (1992) and Makoto and Hiroshi (1999) studied a single machine problem with a machine availability constraint, respectively. Lee and Liman (1993) considered a two-parallel-machine problem where one machine has an availability constraint and the other can process the jobs continuously. Allaouia et al. (2006, 2008) and Cheng and Wang (2000) studied a two machine flowshop scheduling problem with an availability constraints, respectively. Note that all of these results assume that there is only one unavailability or availability period for each machine. However, maintenance activity has been scheduled regularly, or periodically in many manufacturing systems. Recently, Liao and Chen (2003) studied a single machine scheduling problem with periodic maintenance for the objective of minimizing the maximum tardiness. They proposed a branch and bound algorithm to derive an optimal schedule and a heuristic algorithm to search the near-optimal solution for

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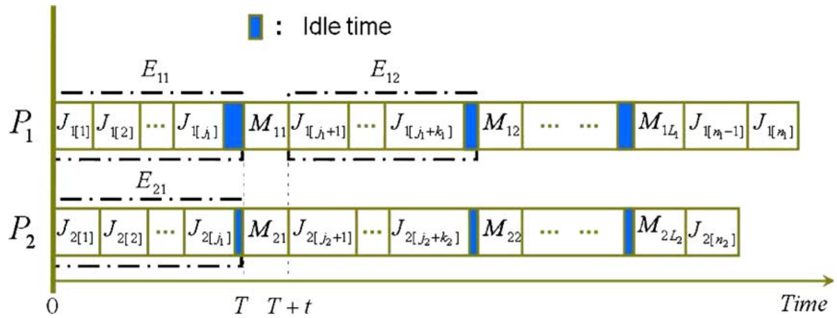

Fig. 1. Gantt chart of schedule with periodic maintenance and non-preemptive jobs.

large-sized problems. In addition, Chen (2006) studied a total flow time minimization problem on single machine with periodic maintenance. Ji et al. (2007) studied the same scheduling model with the objective of minimizing the makespan. They showed that the worst-case ratio of the classical *LPT* algorithm is 2. Further they pointed that there is no polynomial approximation algorithm with worst-case bound less than 2 unless $P=NP$, so the LPT algorithm is the best possible algorithm.

Little research has been done in the literature for the model of jointly scheduling maintenance activities and jobs. Lee and Chen (2000) studied a parallel machine scheduling problem where each machine must be maintained once during the planning horizon. Allaouia et al. (2008) studied a two-machine flow shop scheduling problem for the objective of minimizing the makespan. Qi et al. (1999) and Akturk et al. (1999, 2003, 2004) considered single machine scheduling problems where the maintenance activities need to be scheduled jointly with jobs, respectively. Recently, Qi (2007) considered two scheduling problems with this kind of maintenance activity on single machine and analyzed the worst-case performance of the classical SPT and EDD algorithms, respectively. Xu et al. (2008) studied a parallel machine scheduling problem with almost periodic maintenance requirement to minimize the completion time of the last finished maintenance. They proposed a heuristics algorithm named BFD-LPT for this problem and showed that there is no polynomial time approximation algorithm with a worst-case bound less than 2 unless $P=NP$.

To the best of our knowledge, there is no research on two parallel machines with periodic maintenance or jointly scheduled maintenance in the literature. Thus, in this paper, we study the problems of processing a set of n jobs on two identical parallel machines subject to these two types of maintenance. Two scheduling models are considered. In the first model, the maintenance activities are performed periodically and the objective is to schedule the jobs on two machines such that the makespan is minimized. In the second model, the maintenance activities are determined jointly with the scheduling of jobs, and the objective is to minimize the total completion time of jobs. The remainder of this paper is organized as follows. In the next section, we define the problems formally. In Section 3, we study the makespan minimization problem on two identical parallel machines subject to periodic maintenance. In Section 4, we study the total completion time minimization problem subject to jointly scheduled maintenance. Our focus is to design efficient heuristic algorithms for these two problems and analyze the performance ratio of the algorithms.

2. Problem formulation

The problem can be defined formally as follows: Given a set of n independent jobs $\mathcal{J}=\{J_1,J_2,\ldots,J_n\}$, which are processed on two identical parallel machines P_1 and P_2. All the jobs are simultaneously available at time zero. Each job J_j is assigned a processing time p_j. In the production process, each machine has to be stopped for maintenance after working continuously for a period of time. Suppose that the longest possible consecutive working time units for the machine is T and it takes t time units for each maintenance activity (in order to have a feasible schedule, we need to assume that $\max_{j=1}^{n} p_j \le T$). We focus on the problem in which job preemption is not allowed and the machines are *non-resumable*, i.e., if a job cannot be completed before the maintenance activity, it must be totally restarted. Although T and t mathematically could be any positive numbers, in practice we usually have $T>t$, i.e., the available working time of the machine is longer than the unavailable time. For example, in the factory, a worker can have two days off after working five consecutive days. Thus, in the following, we refer to $t<T$ connotatively.

We will consider two types of maintenance activities. In the first model, the maintenance activities are fixed and performed periodically. An illustration of this model in the form of a Gantt chart is given in Fig. 1, where M_{ij} denotes the j th maintenance on machine P_i (here we assume that each machine just completed a maintenance activity at time $t=0$, i.e., M_{i0} denote the moment $t=0$), E_{ij} denotes the time interval between M_{ij-1} and M_{ij}, and $J_{i[k]}$ denotes the k th job scheduled on machine P_i. In this case, the length of the time interval between any two consecutive maintenance activities is T. Idle time on each machine is unavoidable. Let C_j be the completion time of job J_j. The objective is to minimize the makespan, which is defined as $C_{\max}=\max_{j=1}^{n} C_j$. Using the three-field notation of Graham et al. (1979), we denote this scheduling problem as $P2/nr-pm/C_{\max}$, where $nr-pm$ represents the jobs are non-preemptive and the machines need to be maintained periodically.

In the second model, the maintenance activities are determined jointly with the scheduling of jobs. An illustration of this model in the form of a Gantt chart is given in Fig. 2, where M_{ij}, E_{ij}, $J_{i[k]}$ have the same meanings as that in the Fig. 1. In addition, let T_{ij} denote the total processing time of jobs in E_{ij}, and n_{ij} denote the number of jobs in E_{ij}. In this case, idle time on any machine is not necessary. The objective is to minimize total completion time of jobs. Using the traditional three-field notation, we denote this scheduling problem as $P2/nr-jm/\sum_{j=1}^{n} C_j$, where $nr-jm$ represents the jobs are non-preemptive and the maintenance activities are jointly scheduled with the jobs.

3. The problem $P2/nr-pm/C_{max}$

As we all know, the problem of minimizing the makespan on two identical parallel machines (i.e., $P2\|C_{\max}$) is *NP-hard*, and *LPT* algorithm is an efficient heuristic algorithm with the worst-case performance ratio 7/6. When the periodic maintenance constraint is added, the following theorem holds true.