Available online at www.sciencedirect.com

ScienceDirect

Research in Economics 61 (2007) 165–184

RESEARCH IN ECONOMICS

www.elsevier.com/locate/rie

Interactive epistemology in games with payoff uncertainty

Pierpaolo Battigalli[a,*], Marciano Siniscalchi[b,1]

[a] *Università Bocconi, IEP and IGIER, 5 Via Salasco, 20136, Milano, Italy*
[b] *Economics Department, Northwestern University, 2001 Sheridan Road, Evanston, IL 60208, USA*

Abstract

We adopt an interactive epistemology perspective to analyse dynamic games with partially unknown payoff functions. We consider solution procedures that iteratively delete strategies conditional on private information about the state of nature. In particular we focus on a weak and a strong version of the Δ-rationalizability solution concept, where Δ represents given restrictions on players' beliefs about state of nature and strategies [Battigalli, P., 2003. Rationalizability in infinite, dynamic games of incomplete information. Research in Economics 57, 1–38; Battigalli, P., Siniscalchi, M., 2003. Rationalization and incomplete information. Advances in Theoretical Economics 3 (Article 3). http://www.bepress.com/bejte/advances/vol3/iss1/art3]. We first show that *weak Δ-rationalizability* is characterized by initial common certainty of rationality and of the restrictions Δ, whereas *strong Δ-rationalizability* is characterized by common strong belief in rationality and the restrictions Δ (cf. [Battigalli, P., Siniscalchi, M., 2002. Strong belief and forward induction reasoning. Journal of Economic Theory 106, 356–391]). The latter result allows us to obtain an epistemic characterization of the iterated intuitive criterion. Then we use the framework to analyse the robustness of complete-information rationalizability solution concepts to the introduction of "slight" uncertainty about payoffs. If the set of conceivable payoff functions is sufficiently large, the set of strongly rationalizable strategies with slight payoff uncertainty coincides with the set of complete-information, weakly rationalizable strategies.

Keywords: Interactive epistemology; Rationalizability; Forward Induction; Iterated intuitive criterion; Robustness

1. Introduction

Games with payoff uncertainty are situations of strategic interaction where players' payoff functions are not common knowledge. Following Harsanyi's approach (Harsanyi, 1967–68), such situations are usually studied as Bayesian games. A Bayesian game adds to the primitives of the model (players set, rules of interactions, possible payoff functions, players' private information about such functions) a list of "types" for each player, whereby a type determines (implicitly) a whole hierarchy of beliefs about the unknown payoff parameters. In economic applications the analysis is often simplified by assuming that the correspondence from types to hierarchies of beliefs is trivial. For example, it is often assumed that each player's first-order beliefs, i.e. his beliefs about the payoff parameter vector, say θ, are solely determined by his private information about θ according to some mapping, and that all these mappings are common knowledge.

* Corresponding author. Tel.: +39 0258365402.
E-mail addresses: pierpaolo.battigalli@uni-bocconi.it (P. Battigalli), marciano@northwestern.edu (M. Siniscalchi).
[1] Tel.: +1 847 491 5398.

1090-9443/$ - see front matter
doi:10.1016/j.rie.2007.10.003

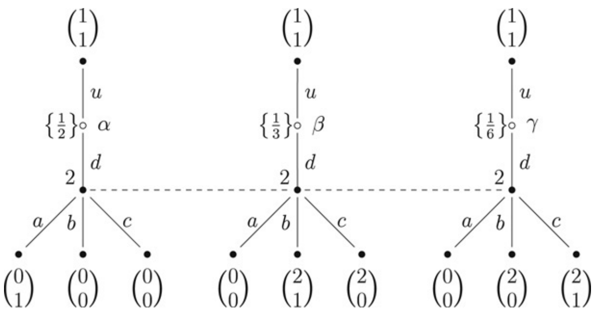

Fig. 1. A signalling game.

The signalling game depicted in Fig. 1 provides an example. Players 1 and 2 move sequentially: Player 1 chooses either u (up), thus terminating the game, or d (down); after d, Player 2 chooses between a, b, or c. There are three possible pairs of payoff functions (mappings from complete sequences of actions to payoffs pairs) indexed by a state of nature $\theta \in \{\alpha, \beta, \theta\}$. Player 1 knows the true θ and Player 2 does not. This is represented by drawing three possible game trees with distinguished roots α, β and γ respectively and joining the three possible decision nodes of Player 2 with a dashed line signifying his ignorance of the true game tree. The model described so far is what we call a game with payoff uncertainty. A very simple way to obtain a Bayesian game is to specify an initial belief of the uninformed Player 2 (e.g. $\Pr_2(\alpha) = \frac{1}{2}$, $\Pr_2(\beta) = \frac{1}{3}$, $\Pr_2(\gamma) = \frac{1}{6}$ as in Fig. 1) and to assume that it is common knowledge that (Player 1 knows the true θ and) Player 2 holds this initial belief. But one could consider much more complicated Bayesian games. For example, if Player 1 were unsure about the initial belief about θ of Player 2, one should introduce several possible types of Player 2, corresponding to possible beliefs about θ, and a belief of Player 1 about such types. Were this belief not known to Player 2, one should multiply the possible types of Player 1, and so on.

Equilibria of Bayesian games are very sensitive to the precise specification of higher-order beliefs (see Weinstein and Yildiz, 2007). This is especially problematic because economic modellers find it hard to provide non- arbitrary specifications of the fine details of hierarchical beliefs.

Battigalli (1999, 2003), Battigalli and Siniscalchi (2003) propose a different approach to the analysis of dynamic games with payoff uncertainty: instead of specifying a (more or less complex) type space *à la* Harsanyi, they suggest to take as given some restrictions Δ on players' initial and updated beliefs about θ and their opponents' strategies, and then iteratively delete private information-strategy pairs that are inconsistent with progressively higher levels of mutual certainty of rationality and of the restrictions Δ. For example, in the signalling game above, the modeller may find it reasonable to assume that $\Pr_2(\beta)$ is larger than $\Pr_2(\gamma)$, and there is common certainty of this fact; Δ would then be the set of beliefs profiles such that $\Pr_2(\beta) > \Pr_2(\gamma)$. The resulting solution concept, called Δ-rationalizability, is therefore parametrized by the assumed restrictions Δ.[2] Battigalli and Siniscalchi (2003) specifically focus on a strong version of Δ-rationalizability, akin to extensive form rationalizability (Pearce, 1984), that also captures a forward induction principle. Indeed, they show that, when Δ reflects agreement of beliefs with a given probability distribution ζ on the terminal nodes of a signalling game, then non-emptiness of the strong Δ-rationalizability solution is equivalent to ζ passing the Iterated Intuitive Criterion of Cho and Kreps (1987). Battigalli (1999, 2003) also considers a weak version of the solution concept that only relies on initial common certainty of rationality and the restrictions Δ. These papers present several examples and economic applications of the approach and report about other applications in the literature.

To illustrate, independently of Player 2's initial beliefs, the signalling game depicted in Fig. 1 has a pooling (sequential) equilibrium where each type of Player 1 chooses u and Player 2 would choose a after d. To see this, note that the best response of Player 2 is the action whose label is the Latin equivalent of the ex-post most likely θ (i.e. a if α is most likely, etc.); since beliefs off the equilibrium path are not determined via Bayes' rule, we may have $\Pr_2(\alpha|d) \geq \max\{\Pr_2(\beta|d), \Pr_2(\theta|d)\}$, making action a a (sequential) best response. This outcome is *a fortiori* weakly Δ-rationalizable for every Δ that allows $\Pr_2(\alpha|d) \geq \max\{\Pr_2(\beta|d), \Pr_2(\theta|d)\}$. However, the pooling-equilibrium outcome is not strongly Δ-rationalizable; this follows from a forward-induction argument. Action d is dominated for

[2] For this reason it has been referred to as an "umbrella solution concept" by Dekel et al. (2007).