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Applications of Geometric Moment Theory Related to Optimal Portfolio Management

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Abstract—In this article, we start with the brief description of the essence of geometric moment theory method for optimization of integrals due to Kemperman [1–3]. Then, we solve several new Moment problems with applications to stock market and financial mathematics. That is, we give methods for optimal allocation of funds over stocks and bonds at maximum return. More precisely, we present here the optimal portfolio management under optimal selection of securities so to maximize profit. The above are done within the models of optimal frontier and optimizing concavity. © 2006 Elsevier Ltd. All rights reserved.

Keywords—Moments and geometric moment theory, Optimal portfolio management, Problem of optimal frontier, Concavity.

1. INTRODUCTION

The main problem we solve here is the optimal allocation of funds over stocks and bonds and at the same time, given certain level of expectation, best choice of securities on the purpose to maximize return. The results are very general so that they stand by themselves as "formulas" to treat other similar stochastic situations and structures far away from the stock market and financial mathematics. The answers to the above described problem are given under two models of investing, the optimal frontier and optimizing concavity, as being the most natural.

There are given many examples all motivated from financial mathematics and of course fitting and working well there. The method of proof derives from the geometric moment theory of Kemperman, see [1–3], and several new moment results of very general nature are presented here. We start the article with basic geometric moment review and we show the proving tool we use next repeatedly.

To the best of our knowledge this paper is totally new in literature as a whole and nothing similar or prior to it in any form exists there. We hope it is well received by the community of

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mathematical-economists and that can be useful there, by giving some definite real answers to existing questions in optimal portfolio theory.

The continuation of this work will be one to derive algorithms out of this theoretical work and create computer software of implementation and work with actual numerical data of the stock market.

2. BACKGROUND

GEOMETRIC MOMENT THEORY. (See [1–3].) Let $g_1, \dots, g_n$, and h be given real-valued $(\mathcal{B} \cap \mathcal{A})$ (Borel and $\mathcal{A}$)-measurable functions on a fixed measurable space $T = (T, \mathcal{A})$. We also assume that all one-point sets $\{t\}$, $t \in T$ are $(\mathcal{B} \cap \mathcal{A})$-measurable. We would like to find the best upper and lower bounds on the integral,

$$\mu(h) = \int_T h(t)\, \mu(dt),$$

given that μ is a (with respect to $\mathcal{A}$) probability measure on T with given moments,

$$\mu(g_j) = y_j, \qquad j = 1, \dots, n.$$

We denote by $m^+ = m^+(T)$ the collection of all $\mathcal{A}$-probability measures on T such that $\mu(|g_j|) < \infty$ $(j = 1, \dots, n)$ and $\mu(|h|) < \infty$. For each $y = (y_1, \dots, y_n) \in \mathbb{R}^n$, consider the bounds $L(y) = L(y \mid h) = \inf \mu(h)$, $U(y) = U(y \mid h) = \sup \mu(h)$, such that

$$\mu \in m^+(T); \quad \mu(g_j) = y_j, \qquad j = 1, \dots, n.$$

If there is not such measure μ, we set $L(y) = \infty$, $U(y) = -\infty$. Let $M^+(T)$ be the set of all probability measures on T that are finitely supported. By the next Theorem 1, we get that

$$L(y \mid h) = \inf\left\{\mu(h) : \mu \in M^+(T),\ \mu(g) = y\right\} \tag{1}$$

and

$$U(y \mid h) = \sup\left\{\mu(h) : \mu \in M^+(T),\ \mu(g) = y\right\}. \tag{2}$$

Here, $\mu(g) = y$ means $\mu(g_j) = y_j$, $j = 1, \dots, n$.

THEOREM 1. *(See [4–6].) Let $f_1, \dots, f_N$ be given real-valued Borel measurable functions on a measurable space $\Omega = (\Omega, \mathcal{F})$. Let μ be a probability measure on Ω such that each f_i is integrable with respect to μ. Then, there exists a probability measure μ' of finite support on Ω satisfying*

$$\mu'(f_j) = \mu(f_j), \qquad \text{for all } j = 1, \dots, N.$$

One can even attain that the support of μ' has at most $N + 1$ points.

Hence, from now on, we deal only with finitely supported probability measures on T. Consequently, our initial problem is restated as follows.

Let $T \neq \emptyset$ set and $g: T \to \mathbb{R}^n$, $h: T \to \mathbb{R}$ be given $(\mathcal{B} \cap \mathcal{A})$-measurable functions on T, where $g(t) = (g_1(t), \dots, g_n(t))$. We want to find $L(y \mid h)$ and $U(y \mid h)$ defined by (1) and (2).

Here, a very important set is

$$V = \operatorname{conv} g(T) \subseteq \mathbb{R}^n,$$

where "conv" means convex hull, and the range,

$$g(T) = \{z \in \mathbb{R}^n : z = g(t) \text{ for some } t \in T\}.$$

Clearly, g is a curve in n-space (if T is an one-dimensional interval) or a two-dimensional surface in n-space (if T is a square).