Available online at www.sciencedirect.com

SCIENCE DIRECT

Fuzzy Sets and Systems 157 (2006) 1317 – 1327

FUZZY sets and systems

www.elsevier.com/locate/fss

A stochastic soft constraints fuzzy model for a portfolio selection problem

V. Lacagnina, A. Pecorella*

Dipartimento di Scienze Statistiche e Matematiche "Silvio Vianelli", Università di Palermo, Viale delle Scienze - 90128 Palermo, Italy

Received 25 February 2005; received in revised form 5 October 2005; accepted 7 October 2005
Available online 4 November 2005

Abstract

The financial market behavior is affected by several non-probabilistic factors such as vagueness and ambiguity. In this paper we develop a multistage stochastic soft constraints fuzzy program with recourse in order to capture both uncertainty and imprecision as well as to solve a portfolio management problem. The results we obtained confirm the studies carried out in literature addressed to integrate stochastic and possibilistic programming.

Keywords: Stochastic programming; Fuzzy programming; Portfolio management

1. Introduction

The largest part of mathematical models used in many areas of decision making has primarily a *hard* or *crisp* structure, i.e. the solutions are considered to be either feasible or unfeasible. This dichotomy often forces researchers to represent real behaviors by *yes-or-not* models unable to cover uncertainty, complexity, and vague or imprecise concepts and variables. This is particularly true if the problem includes: (a) vaguely defined relationships, human evaluations, uncertainty due to inconsistent or incomplete evidence; (b) natural language to be modeled; (c) state variables that can be described only approximately. As stated in Zadeh's principle of incompatibility: "*as the complexity of the system increases*, *our ability to make precise and yet significant statements about its behavior diminishes until a threshold is reached beyond which precision and significance* (*or relevance*) *become almost mutually exclusive characteristics*" [20]. The growing interest in both uncertainty and vagueness sources and the different approaches by which they could be *controlled*, has contributed to develop new optimization models most of which have already been applied in many areas and in particular in financial modeling. Sophisticated techniques of stochastic programming and fuzzy theory have been used to solve real portfolio problems [12,8,11, 7,17].

We propose a multistage stochastic soft constraints fuzzy model with recourse to solve a portfolio management problem. The preliminary results confirm the importance of integrating stochastic programming with fuzzy logic in modeling real financial problems.

The remaining sections of this paper are organized as follows. Section 2 is devoted to a brief survey of linear fuzzy programming. Section 3 is dedicated to a brief survey on stochastic programming. Section 4 shows the asset/liability

* Corresponding author. Tel.: +39 091 6626232; fax: +39 091 426781.
E-mail addresses: ricopa@unipa.it (V. Lacagnina), tonpec@unipa.it (A. Pecorella).

0165-0114/$ - see front matter © 2005 Elsevier B.V. All rights reserved.
doi:10.1016/j.fss.2005.10.002

management model whereas Section 5 explains our model. Finally, the empirical analysis is listed in Section 6. Section 7 concludes the paper.

2. Fuzzy programming

Fuzzy optimization overcomes the drawback involved by the application of standard mathematical programming: the set of feasible alternatives must be well defined. In the fuzzy programming both the objective function and the feasible set may be fuzzy. For example, the feasible set is explicitly specified by its corresponding membership function whose values indicate the degrees of feasibility of the particular alternatives. Then, the classical linear programming problem can be fuzzified [19,10] as follows:

$$f(x) = \sum_{i=1}^{n} c_i x_i \lesssim z$$

subject to:

$$\sum_{i=1}^{n} a_{ij} x_i \lesssim b_j, \quad j = 1, 2, \ldots, m,$$

$$x_i \geqslant 0, \quad i = 1, 2, \ldots, n. \tag{1}$$

In model (1)

- the objective function $f(x) = \sum_{i=1}^{n} c_i x_i$ should be *essentially smaller than or equal to* an aspiration level z,
- the constraints $\sum_{i=1}^{n} a_{ij} x_i$ should be *reasonably well-satisfied*.

The meaning of "$\lesssim$" can be formalized as follows. Let H denote the matrix obtained by inserting the vector $[c_i]$ as the first row of the matrix A of technological coefficients a_{ij}, and let $(Hx)_k = \sum_{i=1}^{n} h_{ik} x_i$ define the function

$$f_k((Hx)_k) = \begin{cases} 1 & \text{for } (Hx)_k \leqslant w_k, \\ 1 - \dfrac{(Hx)_k - w_k}{d_k} & \text{for } w_k < (Hx)_k < w_k + d_k, \\ 0 & \text{for } (Hx)_k \geqslant w_k + d_k, \end{cases} \tag{2}$$

where $w^T = (w_1, \ldots, w_{m+1})^T = (z, b_1, \ldots, b_m)^T$, and d_k indicates some subjectively admissible violations of the constraints.

Problem (1) can be written in the following form:

$$\bigwedge_{k=1}^{m+1} f_k((Hx)_k) \rightarrow \max_{x=(x_1,\ldots,x_n)}. \tag{3}$$

Negoita and Sularia [13] proved that problem (3) is equivalent to the following model:

$$\lambda \rightarrow \max_{x=(x_1,\ldots,x_n)}$$

subject to:

$$(\widetilde{H}x)_k - \lambda \leqslant \widetilde{w}_k, \quad k = 1, 2, \ldots, m+1,$$

$$x_i \geqslant 0, \quad i = 1, 2, \ldots, n, \tag{4}$$

where $\widetilde{w}_k = w_k/d_k$ and $(\widetilde{H}x)_k = (Hx)_k/d_k$.