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Regime switching based portfolio selection for pension funds

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Abstract

This paper shows how a mean variance criterion can be applied to a multi period setting in order to obtain efficient portfolios in an asset and liability context. The optimization model allows for rebalancing activities, transaction costs, stochastic volatilities for both assets and liabilities. Furthermore, a general framework for the projection of pension fund liabilities as well as for the generation of asset returns is given. In a further step the dynamics of the liability maturity structure is modeled as customized index, whose volatility and correlation with asset returns become integral components of the applied regime switching approach. The numerical results illustrate the diversification of the assets and its risk return pattern in dependency of the liability dynamics.

JEL classification: G11; G15; G23; G32; C53; C61

Keywords: Portfolio management; Dynamic asset allocation; Stochastic multistage programming; Regime switching; Modeling of pension fund liabilities; Projection of the funding ratio

1. Introduction

The management of a pension fund is especially characterized by its guaranteed long term obligations that are usually protected either by law or by the underlying insurance policy. The funding of the fund's pension plans or rather its liabilities is effected by the

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contributions of the insurees, their sponsoring firm and moreover by the *return* on the invested capital. Due to the long term investment horizon the return on the invested capital becomes a decisive element in funding a pension plan. In a defined benefit framework the return on the invested capital should therefore pace out the *technical interest rate* whereas in a defined contribution framework the return should pace out at least the *guaranteed minimal return*.

Unlike these (usually) guaranteed elements of any retirement plan, the return on the pension fund's asset allocation is *not* guaranteed. The observable stochastic volatilities on financial markets therefore necessitate a suitable Asset and Liability Management (ALM) for pension funds to safeguard the pension claims of the beneficiaries. In this context, the main task of any pension fund manager lies in the sustainable funding of the pension liabilities as well as in securing the payments of benefits to the beneficiaries. Hence, a firm implementation of sophisticated quantitative methods and concepts is indispensable in order to tackle the uncertainties that occur with any committed investment. Prominent models challenging this complex task can be found in the field of stochastic programming; like for example in Cariño et al. (1998), Mulvey and Ziemba (1998), Consigli and Dempster, 1998, Pflug and Swietanowski (2000), Mulvey and Shetty (2004), Drijver (2005), Zenios and Ziemba (2006) or Hilli et al. (in press).

This paper is structured as follows. In Section 2, a sophisticated model to project the liabilities of a pension fund will be briefly sketched. In Section 3, a regime switching model to estimate the expected returns of the pension fund's assets categories will be derived. Section 4 captures the key optimization problem in a way strategic asset allocation process allows to take the liability dynamic into account. It will be shown in Section 5 that the careful attention of the correlation structure between assets and liabilities, by means of the regime switching approach, will have a significant impact on the efficient asset allocation and therefore on the risk profile of the whole pension plan.

2. Modeling pension fund liabilities

The main purpose of a pension fund's asset allocation is to ensure the payments of well defined pension benefits to the insurees at well specified dates. In this connection, a detailed knowledge of the liability structure is a precondition for any successful ALM. In the following, a multi-period model predicting pension fund liabilities (either for defined benefit or defined contribution schemes) will be introduced.

The future development of pension fund liabilities is mainly driven by two factors: the evolvement of the population of insurees as well as by the growth of the insurees' future wages and accordingly their benefits. Concerning the projection of the population, the authors propose a multi-period Markov-chain framework, whereas future wages and benefits are modeled with appropriate stochastic processes. The following Fig. 1 shows the main principle of this approach (see also Koller, 2000).

The actuarial state of an insuree shall be described by the stochastic process X with $X_t(\omega) \in S$. The underlying insurance policy defines for every actuarial state of the insuree the financial obligation due to the pension fund. The future pension fund liabilities are then derived by the so called *contractual functions* that are given by the underlying insurance policy and define for every possible actuarial state of the insuree the resulting financial obligation for the pension fund. In Fig. 1, these contractual functions are labeled with $a_{ij}(t)$, respectively, $a_{ii}(t)$. $a_{ii}(t)$, e.g., describes the function or rather the payment in case the