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Stochastic processes evolutionary spectrum estimation via harmonic wavelets

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Abstract

The problem of estimating the power spectrum of non-stationary stochastic processes using dyadic, generalized, and filtered harmonic wavelets is addressed. Explicit relationships between the statistical moments of the coefficients of the wavelets based representation of the process and of its evolving spectral values are given. It is shown that the popular concept of a separable evolutionary spectrum involving a deterministic modulating envelope is subject to interpretation. Further, mathematical expressions elucidating the analogy between the wavelets based spectral representation and the traditional one involving trigonometric functions are derived. Finally, the applicability and the physical soundness of the developed procedure are demonstrated by applying it to records of the Kocaeli, Turkey earthquake (8/17/1999).

Keywords: Wavelet; Stochastic process; Evolutionary spectrum; Seismic; Non-linear

1. Introduction

The wavelet transform yields a useful representation of a function in the time–frequency domain [3,15]. Recent applications of the wavelet transform to engineering problems can be found in several studies that refer to dynamic analysis of structures, system identification, damage detection, and a plethora of other themes. This study applies the wavelet transform to the problem of estimating the power spectrum of a non-stationary stochastic process; Refs. [11,12,14] provide examples of previous engineering approaches for addressing this important problem.

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Nomenclature

E	mathematical expectation
H	non-normalized evolutionary spectrum
N	signal length
Re	real part of a complex number
S	normalized evolutionary spectrum
S_{KT}	Kanai–Tajimi spectrum
T	sampling period
W	Fourier transform of the harmonic wavelet function
$\widehat{W}$	Fourier transform of the filtered harmonic function
$[W_{\psi}f]$	wavelet transform of function f using a wavelet ψ
f	function to be used in wavelet transform
g	enveloping function used to generate records of a separable process
i	introduces the imaginary part ($\mathrm{i} = \sqrt{-1}$)
j	dyadic wavelet scale
k	wavelet time position
m, n	parameters defining the generalized and filtered harmonic wavelet scale
r	filtered harmonic wavelet time position
t	dimensionless time
$\hat{t}$	time in seconds
w	harmonic wavelet function
α	harmonic wavelet coefficients
ψ	mother wavelet
ζ	damping ratio of the second order filter associated with the Kanai–Tajimi spectrum
φ	oscillatory function
ω	dimensionless frequency
$\widehat{\omega}$	frequency in radians per seconds
ω_0	natural frequency of the second order filter associated with the Kanai–Tajimi spectrum

The notion of the spectrum of a stochastic process can be associated with a trigonometric representation, which involves a decomposition of the process in sines and cosines. In this manner, it is easy to identify the contribution of parts of the process with specific frequency content to the total energy [9]. Other forms of oscillatory functions can be used to represent non-stationary processes and to capture the change in time of the probabilistic characteristics of the process [10]. For instance, the Wigner–Ville time–frequency analysis can be used [4]. However, the Wigner–Ville representation of the spectrum lacks, to a certain degree, physical meaning since it yields negative values for the spectrum in several cases. Similar difficulties are encountered in using other joint time–frequency analysis schemes. Nevertheless, wavelets, which are oscillatory functions of zero mean and of finite energy, can be used to obtain a rigorously defined and physics-compatible time–frequency representation of a stochastic process [1,2].

In this paper, harmonic wavelets which possess the appealing property of non-overlapping Fourier transforms are used for capturing the evolving spectral content of non-stationary processes. Explicit mathematical expressions elucidating the spectral representation and efficiency aspects of the various members of the harmonic wavelets family are given. Numerical examples involving an analytical model of a non-stationary process derived as the product of a modulating envelope and of a stationary process, and recorded data from a seismic event in Turkey are given.