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Fuzzy Sets and Systems 155 (2005) 215–235

FUZZY sets and systems

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Approximation of stochastic processes by T–S fuzzy systems☆

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Received 8 May 2004; received in revised form 14 March 2005; accepted 5 April 2005
Available online 25 April 2005

Abstract

Fuzzy systems can provide us with universal approximation models of deterministic input–output relationships, but in the stochastic environment few achievements related to the subject have so far achieved. In the paper a novel stochastic Takagi–Sugeno (T–S) fuzzy system is introduced to represent approximately existing randomness in many real-world systems. By recapitulating the general architecture of the stochastic T–S fuzzy rule-based system, we analyze systematically approximating capability of the stochastic system to a class of stochastic processes. By the canonical representation of the stochastic processes, the stochastic fuzzy system is capable of with arbitrary accuracy providing the approximation to the stochastic processes in mean square sense. Finally, an efficient algorithm for the stochastic T–S fuzzy system is developed. A simulation example demonstrates how a stochastic T–S fuzzy system can be constructed to realize the given stochastic process, approximately.

Keywords: Stochastic T–S fuzzy system; Universal approximator; Stochastic process; Canonical representation; Stochastic integral

1. Introduction

The fuzzy systems including T–S systems and Mamdani systems are capable of approximating a wide class of functions, such as continuous functions and integrable functions [3,12,20,22–25], and so on. Like artificial neural networks [4], the approximation research is of much theoretic importance as it can

☆ Project supported by grants from National Natural Science Foundation of China (No. 60375023, No. 60474023, No. 69974041).
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0165-0114/$ - see front matter © 2005 Elsevier B.V. All rights reserved.
doi:10.1016/j.fss.2005.04.002

enrich approximation theory [3,12,24], also of practical importance as it has found very useful in many applied areas, e.g., system identification [17,20], automatic control [3,20], system modelling [17,25], pattern recognition [20] and telecommunication [9], etc.

Although above universal approximation property of fuzzy systems can guarantee their ability for modeling deterministic complex and uncertain systems, such a superior trait may be degraded by existence of randomness—a statistical uncertainty. It should be noted that many practical systems, such as financial markets [2], weather forecast models and control models [14] etc., operate in environment in which includes different types of uncertainty, especially randomness and fuzziness.

Since the mid-1980s, research on the properties of articial neural networks in a stochastic environment has attracted attention from many scholars. Hinton et al. [7] built the Boltzmann machine in 1985 based on Statistical Physics, in which the neuron states evolute with given probability distributions. The network model works like a simulated annealing procedure, and it serves as a simulator of probability distributions [7]. In 1989, Gelenbe [8] put forward a random neural network to simulate biophysical neural behavior, in which the neurons exchange positive and negative impulse signals with given probability distribution. To build efficient data classifying techniques Specht [16] developed in 1990 another stochastic neural network model—probabilistic neural network (PNN). It operates by defining a probability density function to constitute a nonlinear, nonparametric pattern recognition algorithm. Based on Bayes' classifying method PNN puts the statistical kernel estimator into the framework of radial basis function networks. The performance of a PNN is often superior to other state-of-the-art classifier. However, all those stochastic neural networks, Boltzmann machine-like networks, Gelenbe's models and PNN's cannot be used as direct tools to approximate or simulate stochastic processes with some metric senses. This problem begins to attract some scholars' attention, for example, authors in [1,19] have proven that the approximation identity neural networks [5,18] can with mean square sense approximate a class of stochastic processes to arbitrary degree of accuracy. But the fuzziness in real-world systems leads also to insufficiency of the neural network approach to model real input–output processes.

Above facts bring about the motivation of integrating randomness and fuzziness as a useful system model to deal with different types of uncertainty. The stochastic fuzzy system can bridge the gap between randomness and fuzziness, and thus it can simultaneously handle data information and linguistic information, in which statistical uncertainty can arise [11,14,15]. An important and meaningful problem related is to analyze the approximating capability of stochastic fuzzy systems and employ them to model real stochastic processes. In this paper, we concentrate on stochastic T–S fuzzy systems for this purpose and use the stochastic T–S fuzzy systems directly to simulate stochastic processes.

The paper is organized as follows. In Section 2, we present a novel reasoning scheme—stochastic T–S fuzzy rule incorporating randomness with fuzziness in an inference rule as 'IF...THEN...', and a general architecture of T–S stochastic fuzzy rule–based system, i.e. a stochastic T–S fuzzy system is recapitulated, also some useful properties of the system is summarized. Section 3 recalls the theory of stochastic processes, and the 'canonical representation' of a class of processes, which is of central importance to the approximation analysis. Section 4 deals with the approximation of the stochastic T–S fuzzy system to stochastic processes. If the processes are uniformly continuous with mean square sense, the approximation with arbitrary degree of accuracy is analyzed by the processes themselves; If the processes possess the regular representation form, the covariance functions of the processes and the stochastic integrals are the key tools for the discussions of the approximation procedures. In Section 5,