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Stochastics and Statistics

On a scheme for predictive maintenance

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Abstract

An operating system contains a replaceable unit whose wear (i.e. accumulated amount of damage) can be observed over time. When the wear reaches a certain level the unit is no longer able to function satisfactorily and needs to be replaced. Although units are produced to the same nominal specification there is still some random variation among them in their wear rates. This will be expressed by incorporating a random effect, or frailty term, in the model for individual degradation. There are costs for observing the wear on a unit, for replacing a unit, and for allowing a unit to fail before being replaced. When the last cost is comparatively large replacement before failure is preferable. For some standard examples of wear processes the lifetime distributions are obtained and the cost consequences of particular maintenance schemes are investigated.

Keywords: Damage process; Degradation process; Frailty; Gamma process; Growth curves; Lifetime prediction; Predictive maintenance; Random effects; Reliability; Renewal process; Replacement strategy; Wear process; Wiener process

1. Scenario

An operating system contains a replaceable unit whose wear (i.e. accumulated amount of damage) can be observed over time. When the wear reaches a certain level the unit is no longer able to function satisfactorily and needs to be replaced. Various authors have considered this framework in the situation where the degradation or wear of a unit can be measured only at periodic inspection times. Since it is generally less costly to replace a unit before it has failed (i.e. before it is unable to function satisfactorily), maintenance schemes have been proposed in which units are inspected at specified times, with replacement occurring

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as soon as the degradation is observed to have exceeded a specified level (e.g. Park, 1988; Grall et al., 2002).

Most authors have assumed that the degradation processes for separate units are independent and identically distributed. In practice, although units are produced to the same nominal specification there is often still some random variation among them in their wear rates. Thus, the amount of wear at any time varies from one unit to another. This can be expressed by incorporating a random effect, or frailty term, in the model for individual degradation. Many types of stochastic models for degradation have been considered in the literature; see, for example, Singpurwalla (1995) and Grall et al. (2002) for comments and references. The possibilities for mathematical or numerical computation of failure time distributions and replacement strategy characteristics vary according to the model. Several authors have considered independent-increments processes for $Y(t)$, the wear level at time t after installation of the unit. We do likewise, investigating gamma and Wiener processes in particular. Our analysis differs from those of earlier papers (e.g. see Park, 1988 and Grall et al., 2002, for gamma processes) in that we allow for unit-to-unit heterogeneity in degradation paths through the incorporation of random effects. There are costs for observing the wear of a unit, for replacing a unit, and for allowing a unit to fail before being replaced. When the last cost is comparatively large replacement before failure is preferable. Grall et al. (2002) and Rausand and Hoyland (2004, Section 9.6.3), refer to other work with this type of cost structure.

The following terms and notation will be used in this paper. The wear level at time t after installation of a unit is denoted by $Y(t)$ or Y_t. The wear trajectories are taken to have an upward trend, though not necessarily monotonically increasing, starting at $Y_0 = 0$. A unit fails when $Y(t)$ first reaches a specified level, $y^F > 0$, at time T, say; T is the first-passage time of the process $\{Y_t\}$ to y^F. We assume that inspection does not change the condition of a unit or, in particular, its lifetime, and that replacement of a unit takes no time, i.e. no system 'down-time' is incurred. The cost of inspecting a unit is c_1, the cost of replacing a unit is c_2, and the cost of unit failure is c_3. The relative sizes of the costs are taken to be c_1 (small to moderate), c_2 (moderate), and c_3 (large); c_3 is large because failure of a unit in operation results in significant 'down-time' of the system.

The following strategy will be investigated in which a unit is inspected at a specified time t_1 after its installation and replaced if its degradation $Y(t_1)$ at that time has reached a specified value y^C. Otherwise, if $Y_1 < y^C$, a future replacement time $R = \rho(Y_1; t_1, y^C)$ is scheduled; thus, $R \geqslant t_1$ and, typically, R will be a decreasing function of Y_1. In the event that the unit fails unexpectedly, before time t_1, or after time t_1 but before time R, there are two common replacement policies: either (RP1) replacement is deferred until the following inspection time (t_1 or R), or (RP2) it is replaced immediately upon failure. Scheme RP1 tends to occur with standby units whose failure can go undetected until an inspection, e.g. batteries in a smoke alarm; for non-monotone-increasing processes, failure will have occurred by time t_1 if $\sup_{s \leqslant t_1} Y_s \geqslant y^F$. Scheme RP2 is relevant to continuously-working units whose failure causes an immediate system breakdown. Examples of both replacement policies will be given below.

For some standard examples of wear processes the cost consequences of the particular maintenance schemes will be investigated. Also, choice of t_1 and y^C to minimise cost will be examined.

2. Distribution and cost of replacement-cycle times

Let S denote the length of a single replacement cycle, i.e. the time from just after installation of a unit until just after its replacement. In each replacement cycle one of the following four (exclusive) events occurs.

$E_1 = \{T \leqslant t_1\}$: unit fails at or before time t_1, leading to 'system down'.
$E_2 = \{y^C \leqslant Y_1 < y^F, T > t_1\}$: unit is unfailed at time t_1 but replaced because wear level has reached y^C.