

On optimal payment time for a retailer under permitted delay of payment by the wholesaler[☆]

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Abstract

Jamal et al. [2000. Optimal payment time for a retailer under permitted delay of payment by the wholesaler. *International Journal of Production Economics* 66, 59–66] study a wholesaler–retailer supply chain where the retailer is given a permissible credit period to pay back the dues without paying any interest to the wholesaler. The problem is modeled as a cost minimization problem with two decision variables: the payment time P of the retailer and the length of the inventory cycle T . While the model represents, in general, an interesting problem, we show in this note that there are a number of nontrivial flaws contained in the development of their model and therefore the solution they derived is not correct. We give the correct model and derive the corresponding optimal solution.

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1. Introduction

The problem studied by Jamal et al. (2000) can be described as follows: A retailer orders a product from a wholesaler, which is perishable at a deterioration rate θ . Assume that the order quantity is Q , and the demand for the product has a constant rate D . Then, at the beginning $t = 0$, the inventory level is Q , and at $t = T$ the inventory level reduces to zero, where T is called the length of the inventory cycle. It is further assumed that there is a permitted credit period $[0, M]$, during which no interest is

incurred for the retailer. After that, the retailer must pay for any unpaid balance at an interest rate I_p . The retailer can also use the interest earned in the period $[0, M]$ (with a known interest rate I_e) to pay off the wholesaler.

When is the best time P for the retailer to complete the payment to the wholesaler, so that his overall cost is minimized? The main purpose of Jamal, Sarker and Wang's paper is to address this question. This is an interesting problem, which models a significant situation in management of wholesaler–retailer supply chains with perishable products. An optimal solution for the problem may provide not only a desirable strategy for the retailer, but also a deeper understanding on other important issues such as development of wholesale–retailer contracts in such a situation. This line of research has received some attention in the literature. For

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example, problems with permissible delay in payments of retailers have been considered by Aggarwal and Jaggi (1995), Chung et al. (2005), Goyal (1985), Arcelus et al. (2002), Hwang and Shinn (1997), and Sarker and Pan (1994), Kim et al. (1997) have investigated the problem of developing an optimal credit policy to maximize the wholesaler’s profit.

Jamal et al.’s (2000) paper first argues that the total cost for the retailer is represented by a function $TV C(P, T) = TC(P, T) \times T$, and then suggests an iterative search approach to compute the optimal solution (P, T) that minimizes $TC(P, T)$. Unfortunately, there are a number of nontrivial technical flaws contained in their development. In the following sections, we will show that:

- (i) their claim (lines 6–12, left column of p. 63) that the total cost function $TC(P, T)$ is convex in P and T is not correct, and $TC(P, T)$ is actually a saddle function with respect to P and T . Consequently, the payment time P they obtained is in fact the maximum point of $TC(P, T^*)$ instead of the minimum point;
- (ii) they overlooked the possibility that the interest cost to be paid by the retailer may become negative, which implies a pathological scenario that the wholesaler pays interest to the retailer; and
- (iii) the deteriorated value of the perishable product is missed in the development of the interest cost function P_T , and therefore the solution derived based on this function is not valid.

We will correct these mistakes and derive the optimal solution. Numerical results will also be reported.

2. On the convexity of the cost function $TC(P, T)$

Jamal et al. (2000) derived the following two differential equations (refer to Jamal et al. (2000) for the definitions of the notation):

$$\frac{\partial TC(P, T)}{\partial P} = \frac{I_p D c}{T} \left(\frac{1}{\theta} (e^{\theta(T-P)} - 1) - (S/c - 1) P - S I_e M^2 / 2c + S(T - P) I_e / c I_p \right) = 0 \tag{1}$$

and

$$\begin{aligned} \frac{\partial TC(P, T)}{\partial T} &= \frac{cD}{T^2} \left(-\frac{A}{cD} + \frac{\theta + i}{\theta^2} (e^{\theta T} (T\theta - 1) + 1) \right) \\ &\quad - \frac{I_p}{\theta^2} [(e^{\theta(T-P)} - e^{\theta(T-M)})(T\theta - 1) - \theta(P - M)] \\ &\quad + I_p(S/c - 1)(P^2 - M^2)/2 + I_e S(I_p M^2(P - M) - T^2 + M^2 + P^2)/2c = 0. \end{aligned} \tag{2}$$

They suggested (see lines 6–12, left column, p. 63, Jamal et al. (2000) that the optimal values of P and T could be obtained by simultaneously solving (1) and (2), since the computational results indicate that the response surface of $TC(P, T)$ is convex in P and T . They proceeded to suggest an optimal seeking procedure to compute P and T .

It is unfortunate that the function $TC(P, T)$ cannot be convex in P . This is analyzed below.

First, note that $M \leq P \leq T$ and $c \leq S$. For any given T , we can see that

$$\frac{\partial^2 TC(P, T)}{\partial P^2} = \frac{I_p D c}{T} (-e^{\theta(T-P)} - (S/c - 1) - S I_e / c I_p) < 0.$$

The negativity of this second partial derivative means that $TC(P, T)$ must be a concave function with respect to P (see Fig. 1). Therefore, for any given T , the total cost $TC(P, T)$ can only be minimized either at M or at T . If there exists $M \leq P^* \leq T$ such that $\partial TC(P, T) / \partial P = 0$, then the cost function $TC(P, T)$ will be maximized at P^* as shown by Fig. 1. This suggests that the optimal payment time P^* found by Jamal et al. could only be the maximal point (the worst solution) instead of the minimal point (the best solution).

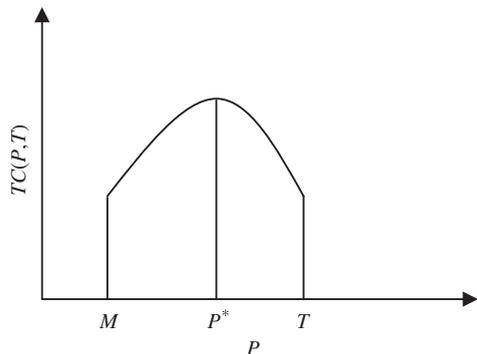


Fig. 1. The cost function of the payment time.

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