Multistep predictions for multivariate GARCH models: Closed form solution and the value for portfolio management

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**A R T I C L E   I N F O**

**Abstract**

This paper derives the closed form solution for multistep predictions of the conditional means and covariances for multivariate ARMA-GARCH models. These predictions are useful e.g. in mean-variance portfolio analysis when the rebalancing frequency is lower than the data frequency. In this situation the conditional mean and the conditional covariance matrix of the cumulated higher frequency returns are required as inputs in the mean-variance portfolio problem. The empirical value of the result is evaluated by comparing the performance of quarterly and monthly rebalanced portfolios using monthly MSCI index data across a large set of GARCH models. Using correct multistep predictions generally results in lower risk and higher returns.

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1. Introduction

This paper derives the closed form solution for multistep predictions of the conditional means and covariances from multivariate GARCH models. These predictions are useful in mean-variance portfolio analysis, when the rebalancing frequency is lower than the data frequency. In the application the empirical value of this result is evaluated in the performance of quarterly rebalanced portfolios based on correct three-step predictions. We compare their performance with that of quarterly rebalanced portfolios incorrectly based on one-step predictions and with the performance of monthly rebalanced portfolios. We use monthly Morgan Stanley Capital International (MSCI) index data for six regions.

Multistep prediction in GARCH models has been considered previously in e.g. Baillie and Bollerslev (1992). They derive the minimum mean squared error forecasts for the conditional mean and the conditional variance of univariate GARCH processes. We extend their results to the multivariate case and derive closed form representations for the conditional mean and the conditional covariances $h$-steps ahead. In addition we derive the explicit formula for the conditional covariance of the sum of the conditional means up to $h$-steps ahead. This corresponds to the conditional variance of the cumulative returns over an $h$-period horizon, when modelling asset returns.

In our empirical application portfolios are adjusted quarterly based on GARCH models estimated with monthly data. This implies that the conditional variances of monthly returns cumulated over three months have to be computed. The empirical part of our study is related to Ledoit, Santa-Clara and Wolf (2003), who apply one-step predictions from multivariate GARCH models for...
portfolio selection using – as we do – MSCI regional indices. However, our study is based on multistep predictions and the results are based on a larger set of GARCH models.

In particular, the value of the derived multistep predictions for portfolio management is evaluated on monthly data for six regional MSCI indices during the evaluation period January 1992 to December 2003. The minimum variance portfolios are tracked for 48 different GARCH models, both for monthly and quarterly rebalancing. In the latter case the quarterly rebalanced portfolios correctly based on multistep predictions and those incorrectly based on one-step predictions are evaluated. We find that using correct multistep predictions generally results in lower risk and higher returns. Furthermore, the correctly computed quarterly rebalanced portfolios exhibit higher returns than monthly rebalanced portfolios.

The paper is organized as follows: In Section 2 the multistep prediction problem is discussed. Section 3 contains the empirical application in portfolio management. Section 4 briefly summarizes and provides conclusions.

2. Multistep prediction in multivariate GARCH models

This section derives the closed form solution for the multistep minimum mean squared error (MSE) prediction of the conditional means, conditional variances and conditional covariances for multivariate GARCH models. Based on these results we also present the generalization of the analogue multistep prediction for univariate GARCH models discussed in Baillie and Bollerslev (1982). Two remarks on the discussion below are in order: First, the limits for prediction horizon \( h \to \infty \) of the results for the minimum MSE predictors of the mean and variance are finite only for stationary processes. Second, the derivations below do not apply to ARCH-in-Mean type models. Detailed derivations are available in our earlier working paper Hlouskova, Schmidheiny and Wagner (2004).

For the derivation of the minimum MSE predictors of \( r_{t+1}, r_{t+1:2+h} \) and their conditional second moments it is convenient to express the ARMA mean Eq. (1) in companion form, compare e.g. Baillie (1987, p. 108):

\[
\begin{bmatrix}
   r_t \\
   r_{t-1} \\
   \vdots \\
   r_{t-p+1} \\
   \varepsilon_t \\
   \varepsilon_{t-q+1}
\end{bmatrix} =
\begin{bmatrix}
   c \\
   A_1 \\
   \vdots \\
   A_p \\
   0 \\
   0 \\
   \vdots \\
   0 \\
   K_i \\
   K_i \\
   \phi
\end{bmatrix}
\begin{bmatrix}
   r_{t-1} \\
   r_{t-2} \\
   \varepsilon_{t-1} \\
   \varepsilon_{t-q+1}
\end{bmatrix} +
\begin{bmatrix}
   0 \\
   0 \\
   0 \\
   0 \\
   0 \\
   0 \\
   0 \\
   \varepsilon_t \\
   \varepsilon_{t-q}
\end{bmatrix}
\]

1 This follows directly from the definition of the one-period returns, calculated as the logarithmic difference of asset prices.

2 Alternatively, the temporal aggregation results of Drost and Nijman (1993), derived for a specific class of univariate GARCH models, can be used to obtain multistep predictions.
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