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Mutual fund competition in the presence of dynamic flows[☆]

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ABSTRACT

This paper analyzes competition between mutual funds in a multiple funds version of the model of Hugonnier and Kaniel (2010). We characterize the set of equilibria for this portfolio management game and show that there exists a unique Pareto optimal equilibrium. The main result of this paper shows that the funds cannot differentiate themselves through portfolio choice in the sense that they should offer the same risk/return tradeoff in equilibrium. This result brings theoretical support to the findings of recent empirical studies on the importance of media coverage and marketing in the mutual funds industry.

1. Introduction

In recent decades, the number of mutual funds offered to investors has grown substantially and now exceeds the number of traded assets in most exchanges (see Gruber, 1996; Massa, 1998), while an increasing number of these mutual funds are operating in the same sector. Most of the funds charge a fraction of fund fees whereby the manager receives a fixed fraction of the assets under management (see Golec, 1999; Golec & Starks, 2004), but the level of these fees varies greatly across funds (see Hortaçsu & Syverson, 2004). As a result, in any given market segment various investment vehicles are offered to the investor in the form of mutual funds which differ in their management fees and, presumably, also in their investment strategies.

The aim of this paper is to investigate if mutual funds competing on the same market can differentiate from each other through portfolio management in a world where investors can move their money in and out of mutual funds. To this end, we study a generalization of the model of Hugonnier and Kaniel (2010) with multiple mutual funds. Specifically, we consider a continuous-time economy populated by a small investor and two mutual fund managers. The small investor implicitly faces high costs that preclude her from trading directly in the equity market. These implicit costs can be related, for example, to the fact that the opportunity costs of spending her time in stock-trading related activities are high. For example, one might think that actively trading multiple risky securities requires considerably more attention than trading in one or two mutual funds. While the investor is precluded from holding equity directly, she is allowed to dynamically allocate money between the two mutual funds and a riskless asset. We impose the natural restriction that the investor cannot short the funds and assume that both funds charge fraction-of-fund fees, albeit at different rates.

To focus on the competition between the funds, while maintaining a tractable setup, we make a few simplifying assumptions. First, agents have complete information and observe the actions of each other.[1] Second, from the perspective of the funds markets are complete. Third, the investor is assumed to have a logarithmic utility function. Fourth, the fund managers are strategic whereas the investor is not. Specifically, when the investor determines her holdings in the funds, she takes the funds' portfolios as given. On the other hand, when a fund manager selects the portfolio of his fund, he takes into account the portfolio of the other and the investors' reaction to the portfolios of the two funds.

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[1] This assumption is rather natural in the context of a continuous-time model. Indeed, if one assumes that market prices are perfectly observed and that the vector of instantaneous risk premia is known to all, then the portfolio strategy of a given fund is public knowledge as soon as its returns are continuously observed.

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In order to solve for the equilibria of the game, we start by studying the investor's utility maximization problem given an arbitrary pair of fund portfolios. Since the investor has logarithmic utility, her optimal strategy depends only on the current characteristics of the funds. In this context, we show that she will invest in both funds, in only one of them or not at all depending on the relative excess returns of the funds with respect to one another. Interestingly, we show that, contrary to the monopolistic case considered in Hugonnier and Kaniel (2010), the investor may find it optimal to invest in a fund whose net-of-fees Sharpe ratio is currently negative. In other words, competition for the investor's money can lead to positive externalities between mutual funds.

In a second step, we take the investor's best response strategy as given and study the Nash game that it induces between the managers. Combining traditional optimization techniques with a change of measure argument we characterize the set of equilibria for this game and show that each of these gives rise to an equilibrium for our delegated portfolio management game. Furthermore, we show that among these equilibria there exists a unique Pareto optimal equilibrium in which the funds offer the same risk/return trade-off. This implies that the investor is indifferent between the two funds in equilibrium and hence that competition does not benefit the investor. In particular, we show that the total fraction of her wealth that the investor will delegate is independent of the funds characteristics, and that its allocation among the funds is arbitrary. This indeterminacy creates a role for marketing in the mutual fund industry and corroborates the findings of recent empirical studies showing the importance of advertising and media coverage, see Barber, Odean, and Zheng (2005), Gallaher, Kaniel, and Starks (2006), Hortaçsu and Syverson (2004), Jain and Wu (2000) and Kaniel, Starks, and Vasudevan (2007) among others.

Fraction-of-fund fees are by far the predominant compensation contract in the mutual fund industry. However, some funds have a performance component in their compensation contract. Basak, Pavlova, and Shapiro (2007), Carpenter (2000) and Grinblatt and Titman (1989) among others, have studied the optimal portfolio strategy of a manager receiving convex performance fees in a setting where the manager receives an exogenous amount of money to manage at the initial date. An analysis of the equilibrium asset pricing implications of both fulcrum fees and asymmetric performance fees is conducted in Cuoco and Kaniel (2001). In that paper both fund managers and unconstrained investors trade directly in equity markets, but investors who use the fund services make allocation decisions only at the initial date. High water mark fees, used in the hedge fund industry, are discussed in Goetzmann, Ingersoll, and Ross (2003). Note however that all these models consider the case of a single mutual fund, and hence abstract from the strategic aspects of competition among mutual funds.

Since the focus of this paper is the impact of dynamic flows on the competition between mutual funds, we take the fee structures of the funds as given. However, it is important to emphasize that we are not taking a stance on whether fraction-of-fund fees is the optimal compensation contract, but instead rely on its widespread use as the motivation for our analysis. Papers that analyze these optimal contracting issues include Carpenter, Dybvig, and Farnsworth (2010), Das and Sundaram (2002a,b), Lynch and Musto (1997), Ou-Yang (2003) and Roll (1992) among many others.

The remainder of the paper is organized as follows. In Section 2, we describe the economic setting, the financial market and the dynamics of the mutual funds. In Section 3, we introduce the players and their objective functions. Section 4 describes the game and defines the notion of equilibrium that we use in this paper. In Section 5, we solve for the best responses of the investor and the managers. In Section 6, we obtain the equilibrium and discuss the impact of imperfect competition between mutual funds. Section 7 concludes. All proofs are deferred to the Appendix.

2. The model

We consider a continuous-time economy with a finite horizon $[0, T]$. The uncertainty is represented by a filtered probability space $(\Omega, \mathcal{F}, \mathbb{F}, P)$ on which is defined a standard n-dimensional Brownian motion represented by the column vector B. The filtration $\mathbb{F} = \{\mathcal{F}_t : 0 \le t \le T\}$ is the usual augmentation of the filtration generated by the Brownian motion and we let $\mathcal{F} = \mathcal{F}_T$.

In the sequel, all processes are assumed to be adapted to $\mathbb{F}$ and all statements involving random quantities are understood to hold either almost surely or almost everywhere. We shall also make use of the following vectorial notation: a $'$ denotes transposition, $\|\cdot\|$ denotes the usual Euclidean norm in $\mathbb{R}^n$ and $\mathbf{1}_k$ is a k-dimensional vector of ones.

2.1. Securities

There is a single perishable good (the numéraire) in units of which all quantities are expressed. The financial market consists of $n+1$ long-lived securities. The first of these is a locally riskless asset whose price S^0 is given by

$$S_t^0 = e^{rt} \tag{1}$$

for some constant interest rate r. The remaining n assets are risky and are referred to as the stocks. The price S_t^i of a share of the i-th stock evolves according to

$$S_t^i = S_0^i + \int_0^t S_s^i \left(a_i ds + \sigma_i' dB_s\right), \tag{2}$$

for some drift a_i and some volatility vector $\sigma_i \in \mathbb{R}^n$ which are both assumed to be constant. We let $a \in \mathbb{R}^n$ denote the column vector of stock drifts, $\sigma \in \mathbb{R}^{n\times n}$ denote the square matrix obtained by stacking up the individual stock volatilities and we assume that σ is invertible.

The assumptions imposed on the coefficients of the model imply that the relative risk premium, or market price of risk, $\xi := \sigma^{-1}(a - r\mathbf{1}_n)$ is well defined. As a result, the formula

$$\left.\frac{dQ}{dP}\right|_{\mathcal{F}_t} = M_t := \exp\left(-\xi' B_t - \frac{1}{2}\|\xi\|^2 t\right) \tag{3}$$

defines an equivalent *risk-neutral probability measure*. Since the volatility matrix of the stocks is invertible this risk-neutral probability measure is uniquely defined and it follows that the financial market is dynamically complete in the absence of trading constraints.

2.2. Mutual funds

We consider two mutual funds, indexed by $i \in \{1, 2\}$, both of which have access to the $n+1$ securities described above. The management fees are assumed to be withdrawn continuously from fund i at the constant rate γ_i applied to the market value of the assets under management and we denote by $\gamma = (\gamma_1, \gamma_2)'$ the vector of instantaneous fee rates.

A trading strategy for fund i is a vector process θ_i specifying the share of the fund's assets invested in each of the stocks. Given such a trading strategy, the return on investments in fund i evolves according to

$$\begin{aligned} dF_{it} &= \left(1 - \theta_{it}'\mathbf{1}_n\right) F_{it}\frac{dS_t^0}{S_t^0} + \sum_{k=1}^n F_{it}\theta_{it}^k \frac{dS_t^k}{S_t^k} - \gamma_i F_{it} dt \\ &= F_{it}\left(r - \gamma_i + \psi_{it}'\xi\right) dt + F_{it}\psi_{it}' dB_t, \end{aligned} \tag{4}$$

where $\psi_i = \sigma'\theta_i$ is the corresponding fund-volatility process. Since the volatility matrix is invertible there is a one-to-one correspondence between fund trading strategies and fund