

# A deteriorating two-system with two repair modes and sojourn times phase-type distributed

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Received 11 October 2003; received in revised form 25 October 2003; accepted 6 December 2003

Available online 2 February 2005

## Abstract

We study a two-unit cold standby system in steady-state. The online unit goes through a finite number of stages of successive degradation preceding the failure. The units are repairable, there is a repairman and two types of maintenance are considered, preventive and corrective. The preventive repair aims to improve the degradation of a unit being operative. The corrective repair is necessary when the unit fails. We will assume that the preventive repair will be interrupted in favour of a corrective repair in order to increase the availability of the system. The random operational and repair times follow phase-type distributions. For this system, the stationary probability vector, the replacement times, and the involved costs are calculated. An optimisation problem is illustrated by a numerical example. In this, the optimal degradation stage for the preventive repair of the online unit is determined by taking into account the system availability and the incurred costs.

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*Keywords:* Phase-type distribution; Markov process; Markovian degradation; Repairable system; Preventive maintenance; Corrective maintenance; Preemption; Availability; Rate of occurrence of failures

## 1. Introduction

Maintenance and costs of reliability systems are topics of interest in industry and engineering. Many papers have been written optimizing different performance measures, costs, and other useful quantities. Different repair/maintenance policies have been introduced in order to solve optimization problems. In general, the systems are continuously deteriorating by many causes: corrosion, fatigue, ageing, and shocks, among others. Trying to fit the models to the real systems, the idea of degradation has been incorporated to the models, and it has been following different ways. When many of these concepts occur simultaneously in the observed systems, the probabilistic models that simulate their evolution are increasingly complicated, and this complexity implies the introduction of the stochastic processes for analyzing them. Among the different stochastic processes that are fitted to the reliability systems,

the Markov ones are probably the most used class; a reason for this may be the possibility of applicability of the results. We show that it is possible to modelize a complex system.

In the present paper we consider a two-system with degradation that is modeled by states that can be classified in good and no-good, and two types of repair depending on the degrading level. Preemption is included in the repair structure, so, it is possible that the system will be operational more time. Moreover, we optimize the availability and the costs involved in the maintenance system. This is a complex system, and extends other previously considered in the literature. The system will be considered in stationary regime.

We extend the concept of degradation given in Van der Duyn Schouten et al. [10], these authors consider that the online unit evolves following an increasing Markov process. The generalization of this concept is performed considering phase-type distributions, so the evolution of the units can be presented in a more versatile way. The operational states are divided in phases representing the operational stages of the units.

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We assume that the operational and repair times follow phase-type distributions, and that the time for interchanging the units between repair and operational channels are negligible. Phase-type distributions were introduced by Neuts [6], and they have proven to be appropriate for modeling the evolution of the units, as can be seen in Neuts and Meier [7]. Faddy [4] considered phase-type distributions for describing the ageing process with movement to the failure states. Yeh [11] studied a deteriorating system optimizing the sojourn times in states involving phase-type distributions. Chandrasekar et al. [3] studied an  $n$ -system modeled by a Markov process whose generator was given by blocks. Neuts et al. [8] considered a unit system governed by phase-type distributions, submitted to different failures with a replacement policy  $N$ , calculating the availability and the rate of occurrence of the different types of failure. This last model has been extended to the semi-Markov by Pérez-Ocón et al. [9]. Given that the phase-type distributions are dense in the class of distributions defined on the positive axis, the study can be considered as an approach to the study of a general system for phase-type distributions.

Two types of repairs are considered: preventive and corrective, depending on the degradation level. The mechanism of preemption is introduced, being the preventive repair of a unit interrupted if the other one needs a corrective repair. In addition, an optimization problem involving costs for the different operations to which the system is submitted is studied. Previous papers related to optimization problems are, for example, Zhang [12] and Barata et al. [2]. We optimize the system maintenance costs in terms of the associated ones to the different operations undergone by the system fixing the availability by means of numerical examples, and determine the optimal preventive repair limit in this model, that is, the state from which the unit can be preventively maintained.

Initially, a unit is online and the other in cold standby. The online unit goes through different degrading stages that can be well differentiated: good and middling stages, and there is a failure stage. There is a repairman and two types of repair can be performed: preventive and corrective, depending on the type of state from which the unit arrived to the repair. The preventive repair occurs when the repairman is idle and the unit online operates in degrading middling stages, being the other unit in standby. The corrective repair occurs when the unit online reaches the failure stage. The system is down only if both units have undergone a failure and they are in corrective repair or waiting for it. Furthermore, we consider that a preventive repair will be interrupted in favour of a corrective one, and so the unit in preventive repair will be in a better condition than before its preventive repair one. Introducing the possibility of preemption, the system will continue operational more time, and the non-operational time of the system is minimized. This maintenance policy can be useful in systems that need to maximize the operational

time even in not good conditions, such as emergency systems. Moreover, the benefits associated to operational states will increase with respect to systems without preemption.

The availability and the rate of occurrence of failures of the units and the system are explicitly determined, in terms of the stationary probability vector. We optimize the system maintenance costs in the numerical application. In this, we consider the example in Van der Duyn Schouten et al. [10], that involves exponential and Erlangian distributions, these are phase type-distributions.

**Definition 1.** The distribution  $H(\cdot)$  on  $[0, \infty[$  is a phase-type distribution (PH-distribution) with representation  $(\alpha, T)$ , if it is the distribution of the time until absorption in a finite-state Markov process with generator

$$\begin{pmatrix} T & T^0 \\ 0 & 0 \end{pmatrix},$$

with initial probability vector  $(\alpha, \alpha_{m+1})$ , where  $\alpha$  is a row  $m$ -probability vector. Throughout this paper,  $e$  will denote a column vector with all components equal to one for which the dimension is determined by the context. The matrix  $T$  of order  $m$  is non-singular with negative diagonal entries and non-negative off-diagonal entries. Moreover, it satisfies  $-Te = T^0 \geq 0$ . The distribution  $H(\cdot)$  is given by:

$$H(x) = 1 - \alpha \exp(Tx)e, \quad x \geq 0.$$

Let it be denoted that  $H(\cdot)$  follows a  $\text{PH}(\alpha, T)$  distribution.

For more details about these distributions see Ref. [6]. We will denote by  $e_k$  a column vector of 1's of order  $k$ , and by  $I_k$  the identity matrix of the same order.

The paper is organized as follows. In Section 2, we introduce the model. The infinitesimal generator is constructed in Section 3. The performance measures are calculated in Section 4. The costs optimization is studied in Section 5. In Section 6 we present a numerical application and illustrate the results calculated throughout the paper.

## 2. The model

In practice, the use of duplicated redundant systems is frequent, for example, emergency systems in hospitals, alarms, aircrafts, and others. The objective of these is their availability at any moment. The failure of the two units implies a catastrophe, and must be eluded, so usually there is a maintenance team for repairing. A step to warrant the use of one unit at any moment is the preventive repair, that allows to repair an operating not-good unit and replace it for the standby one. We present a system operating in such a way that it will be operational the possible maximum time though the units are not good.

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