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Multi-objective mean–variance–skewness model for generation portfolio allocation in electricity markets

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ABSTRACT

This paper proposes an approach for generation portfolio allocation based on mean–variance–skewness (MVS) model which is an extension of the classical mean–variance (MV) portfolio theory, to deal with assets whose return distribution is non-normal. The MVS model allocates portfolios optimally by considering the maximization of both the expected return and skewness of portfolio return while simultaneously minimizing the risk. Since, it is competing and conflicting non-smooth multi-objective optimization problem, this paper employed a multi-objective particle swarm optimization (MOPSO) based meta-heuristic technique to provide Pareto-optimal solution in a single simulation run. Using a case study of the PJM electricity market, the performance of the MVS portfolio theory based method and the classical MV method is compared. It has been found that the MVS portfolio theory based method can provide significantly better portfolios in the situation where non-normally distributed assets exist for trading.

1. Introduction

Based on trading protocols, the competitive electricity markets (EMs) essentially consist of energy market (day-ahead, hour-ahead, and real-time balancing market) and several contractual instruments, such as forward and future contracts [1]. Forward and future contracts are similar, but future contracts are exclusively of financial type while forward contracts comprise the physical delivery of the energy. In competitive environment, generation companies (GenCos) are required to devise their own strategies on how to optimally allocate their generation capacities to the different markets for profit maximization. Moreover, while deriving the profit based generation strategies, the GenCos are confronted with volatile electricity prices and other uncertainties like congestion in transmission lines, unscheduled generating unit outages, etc. Therefore, while making the trading decision, GenCos' objective is not only to maximize its profit, but also to manage the associated risks and this problem can be viewed as a portfolio optimization.

In the last decade, the comprehensive studies [2,3] on various aspects of risk assessment and management for GenCos in competitive electricity markets have been conducted. Value at Risk (VaR) has been applied to risk assessment in electricity markets [4,5]. For hedging the spot price risks for market participants, different forward contracts with their valuation are proposed in [6–8]. In EMs, statistical studies of hedging strategies using financial instruments have been demonstrated in [9,10]. Moreover, some research papers [11–13] have also discussed the problem of allocating the generation capacities between the spot market and various contracts. Majority of aforementioned works for electricity portfolio optimization have employed the standard portfolio optimization approach, i.e., mean–variance (MV) formulation [14] which is precisely a first step of portfolio management. The MV model is a bi-criteria optimization problem where a rational portfolio choice is based on trade-off between risk and return.

However, the standard MV model is based on the assumption that each asset's return follows a normal distribution, so that asset returns can be portrayed only by their first (mean) and second (variance) central moments of distributions. But, substantial number of studies in finance sector [15–20] argued that the higher moments cannot be neglected unless there are reasons to believe that the asset returns are symmetrically distributed around the mean. Moreover, they point out the importance of skewness in the portfolio management. On the other end, empirical studies [21–23] in competitive electricity markets provide evidence indicating that, because of high volatility, spot price as well as return series exhibit statistically significant levels of positive skewness. To support this argument, a detail analysis of historical return of the spot market and bilateral contracts in PJM electricity market is presented in this paper. This study shows that because of high volatility in spot price, it follows the positively skewed distribution and therefore, GenCos returns do not exactly follow the normal distribution.

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Looking to the above issues in electricity portfolio managements, this paper is mainly contributing the followings:

- Using mean–variance–skewness (MVS) model, which is an extension of the classical MV portfolio theory, this paper proposed an approach for generation portfolio allocation considering the maximization of both the expected return and skewness while simultaneously minimizing the risk.
- The MVS portfolio theory is competing and conflicting non-smooth three objectives optimization problem. Third central moment is non-concave function and hence, it looks difficult to solve the resulting MVS portfolio optimization problem. Therefore, unlike single objective optimization method being used in the portfolio literature [11,12], this paper proposed a multi-objective particle swarm optimization (MOPSO) based meta-heuristic method to provide Pareto frontier in single run.

This paper is organized as follows. Section 2 provides a brief review of MVS portfolio framework followed by single and multi-objective portfolio optimization formulation. The brief concept of multi-objective optimization along with Pareto-optimal front and MOPSO are presented in Section 3. The proposed MVS based generation allocation modeling is derived in Section 4 and a case study of the PJM electricity market is given in Section 5 to demonstrate the effectiveness of the proposed method. Finally conclusions are drawn in Section 6.

2. Mean–variance–skewness Portfolio framework

A prerequisite to use the mean–variance (MV) framework is either the relevant distribution of asset returns is normally distributed or the utility function is approximated by only the first two moments. As a results MV approach does not take into account the higher moments in order to describe the investor's assessment of the probability distribution. The first moment represents the expected returns. The second and higher central moments characterize the uncertainty associated to the returns. Investors prefer to maximize the odd portfolio moments and to minimize the even ones. All the even moments measure dispersion (thus, volatility) which is undesirable due to increase in the uncertainty of returns. On the other hand, the odd moments express measures of asymmetry and it can be seen as a way to decrease the extreme values on the loss side and increase them on the gains. For example, maximizing positive skewness (positive skewness refers to a right-handed, elongated tail for the density function) may decrease the probability of having negative returns. As a result, investors are in favor of including skewness in portfolio selection problem because it seems that the combinations that result are more accurate and these give the investors a broader idea of how they can benefit from a portfolio.

The mean–variance–skewness (MVS) model first proposed by Konno and Suzuki [18] is a direct extension of the classical mean–variance (MV) portfolio model. The MVS model is most appropriate choice to the situation where the skewness of the return of assets plays significant role in choosing an optimal portfolio. The general MVS model for portfolio selection problem with $N(N \geq 2)$ risky assets can be described as follows. Let $\boldsymbol{w}_p$ and $\boldsymbol{R}$ denote, respectively, the $(N \times 1)$ vector of proportionate weight and expected returns of the N risky assets in the portfolio p. $\boldsymbol{\Omega}$ and $\boldsymbol{\Lambda}$ represent the non-singular $(N \times N)$ variance–covariance matrix and the $(N \times N^2)$ skewness–coskewness matrix of the N risky asset returns, respectively. The first (mean), second (variance), and third (skewness) central moments, respectively, of the return of a given portfolio p are given by:

$$E(r_p) = \sum_{i=1}^{N} w_{pi} E(r_i) = \boldsymbol{w}'_p \boldsymbol{R} \tag{1}$$

$$\sigma^2(r_p) = \sum_{i=1}^{N}\sum_{j=1}^{N} w_{pi} w_{pj} \sigma_{ij} = \boldsymbol{w}'_p \boldsymbol{\Omega} \boldsymbol{w}_p = \sum_{i=1}^{N} w_{pi}^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} w_{pi} w_{pj} \sigma_{ij} \tag{2}$$

$$s^3(r_p) = \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N} w_{pi} w_{pj} w_{pk} s_{ijk} = \boldsymbol{w}'_p \boldsymbol{\Lambda} (\boldsymbol{w}_p \otimes \boldsymbol{w}_p)$$
$$s^3(r_p) = \sum_{i=1}^{N} w_{pi}^3 s_i^3 + 3 \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} w_{pi}^2 w_{pj} s_{iij} + \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} \sum_{\substack{k=1 \\ k \neq j \\ k \neq i}}^{N} w_{pi} w_{pj} w_{pk} s_{ijk} \tag{3}$$

where w_{pi} and r_i represent the weight of asset i in the portfolio p and the return on the asset i, respectively. σ_i^2 and σ_{ij} represent the variance of return on the asset i and covariance between the returns of assets i and j, respectively. s_i^3 and s_{ijk} represent skewness of the return of asset i and coskewness between the returns of assets i, j, and k, respectively. The sign $\otimes$ stands for the Kronecker symbol product.

The MVS model is competing and conflicting multi-objective optimization problem. An optimal portfolio should maximize both the expected return and skewness while minimizing the risk associated with the return (i.e., variance) simultaneously, as stated below.

$$(\text{Prob}_1) \begin{cases} \text{maximize} f_1(w_p) = \left[E(r_p)\right] \\ \text{minimize} f_2(w_p) = \left[\sigma^2(r_p)\right] \\ \text{maximize} f_3(w_p) = \left[s^3(r_p)\right] \\ \text{subject to} \sum_{i=1}^{N} w_{pi} = 1, \quad w_{pi} \geq 0 \end{cases} \tag{4}$$

2.1. Single objective optimization formulation

Most of the traditional algorithms reformulate a given multi-objective optimization problem into a single objective-function with the help of weighting factors. Using this approach, classical MV portfolio optimization problem in [11,12] has been solved using quadratic programming with help of risk aversion factor. Similarly, a single objective-function of the above multi-objective programming problem (e.g. Prob_1) can be formed by combining the three objective functions and then the same can be optimized by assigning relative weights to represent the importance of each individual function as given below.

$$(\text{Prob}_2) \begin{cases} \text{minimize} f(w_p) = \left[-\beta_1 \left[E(r_p)\right] + \beta_2 \left[\sigma^2(r_p)\right] - \beta_3 \left[s^3(r_p)\right]\right] \\ \text{subject to} \sum_{i=1}^{N} w_{pi} = 1, \quad w_{pi} \geq 0 \end{cases} \tag{5}$$

where β_1, β_2 and β_3 represent investor's relative preference for expected return, risk and skewness, respectively. The Prob_2 is a constrained nonlinear programming problem and generally solved using nonlinear programming techniques. Classical optimization methods like goal programming [19] and linear programming [20], have been used to solve the above problem. However, in order to make this method working, an apriori assumption of the relative importance of each objective has to be incorporated. This makes the