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Particle Swarm Optimization (PSO) for the constrained portfolio optimization problem

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ABSTRACT

One of the most studied problems in the financial investment expert system is the intractability of portfolios. The non-linear constrained portfolio optimization problem with multi-objective functions cannot be efficiently solved using traditionally approaches. This paper presents a meta-heuristic approach to portfolio optimization problem using Particle Swarm Optimization (PSO) technique. The model is tested on various restricted and unrestricted risky investment portfolios and a comparative study with Genetic Algorithms is implemented. The PSO model demonstrates high computational efficiency in constructing optimal risky portfolios. Preliminary results show that the approach is very promising and achieves results comparable or superior with the state of the art solvers.

1. Introduction

Portfolio management is one of the most studied topics in finance. The problem is concerned with managing the portfolio of assets that minimizes the risk objectives subjected to the constraint for guaranteeing a given level of returns. This paper deals with the mean–variance portfolio selection, which is formulated in a similar way as Markowitz did (Elton, Gruber, & Padberg 1976; Markowitz, 1952; Steinbach, 2001). Markowitz introduced the concepts of Modern Portfolio Theory (MPT). His theory has revolutionized the way people think about portfolio of assets, and has gained widespread acceptance as a practical tool for portfolio optimization. But in some cases, the characteristics of the problem, such as its size, real-world requirements (Campbell, Huisman, & Koedijk 2001; Gennotte, 1986; Louis, Jason, & Josef, 1999; Perold, 1984; Zhou & Li, 2000), very limited computation time, and limited precision in estimating instance parameters, may make analytical methods not particularly suitable for tackling large instances of the constrained mean–variance model. Therefore researchers and practitioners have to resort to heuristic techniques that are able to find high-quality solutions in a reasonable amount of time.

Due to the complexity and the instantaneity of the portfolio optimization model, applying meta-heuristic algorithms to portfolio selection and optimization is a good alternative to meet the challenge. Some remarkable studies have been presented to solve asset selection problem. Many meta-heuristic techniques (Chang, Meade, Beasley, & Sharaiha, 2000) have been applied in portfolio selection such as Genetic Algorithms, tabu search and simulated annealing for finding the cardinality constrained efficient frontier. Some hybrid techniques (Gaspero, Tollo, Roli, & Schaerf, 2007) have been applied in portfolio management such as local search and quadratic programming procedure. Preliminary results show that the approach is very promising and achieves results comparable or superior to the traditional solvers. Pareto Ant Colony Optimization (Doerner, Gutjahr, Hartl, Strauss, & Stummer, 2004) has been introduced as an especially effective meta-heuristic for solving the portfolio selection problem and compares its performance to other heuristic approaches (i.e., Pareto Simulated Annealing and the Non-Dominated Sorting Genetic Algorithm) by means of computational experiments with random instances. An artificial neural network model with the Particle Swarm Optimization algorithm (Giovanis, 2009) has been applied to portfolio management and shows the flexibility of hybrid models, such as the superiority in forecasting performance, in relation to the traditional econometric methodology, like Ordinary least square and ARCH-GARCH estimations. Fuzzy Analytic Hierarchy Process (AHP) (Tiryaki & Ahlatcioglu, 2009) has been combined with the portfolio selection problem to model the uncertain environments. A hybrid Genetic Algorithm approach (Jeurissen & van den Berg, 2005) has been investigated for tracking the Dutch AEX-index, it focused on building a tracking portfolio with minimal tracking error.

However, these approaches have some shortcomings in solving the portfolio selection problem. For example, fuzzy approach usu-

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ally lacks learning ability (Chan, Wong, Tse, Cheung, & Tang, 2002); Artificial neural network approach has over-fitting problem and is often easy to trap into local minima (Casas, 2001); while as Genetic Algorithms (Alba & Troya, 1999) are applied to harder and bigger problems there is an increase in the time required to converge for finding adequate solutions.

In order to overcome these drawbacks, PSO model is introduced to solve the portfolio selection and optimization problem. PSO is a population based stochastic optimization technique developed in 1995 (Kennedy & Eberhart, 1995). The underlying biological metaphor for developing PSO algorithm is inspired by social behavior of bird flocking or fish schooling. PSO has become a popular optimization method as they often succeed in finding the best optimum by global search in contrast with most common optimization algorithms. In comparison with the dynamic programming, PSO allows the users to get the sub-optimal solution while dynamic programming cannot. It is very important for the portfolio selection and optimization problem.

There are very few studies on PSO, especially all most none of them deal with the performance comparison with other approaches for solving portfolio optimization problems. The main contribution of this study is to employ a PSO algorithm for portfolio selection and optimization in investment management. Asset allocation in the selected assets is optimized using a PSO based on Markowitz's theory. Using the PSO, an optimal portfolio can be determined. The rest of the paper is organized as follows. Section 2 describes models for portfolio optimization. In Section 3, the background of PSO and previous work are summarized. The PSO model for optimal portfolio is also discussed. In order to test the efficiency of the proposed PSO solver, a simulation and comparative study is performed in Section 4. Final conclusions and future research are drawn in Section 5.

2. Models for portfolio optimization (PO)

One of the fundamental principles of financial investment is diversification where investors diversify their investments into different types of assets. Portfolio diversification minimizes investors' exposure to risks, and maximizes returns on portfolios. It can be referred to as a multi-objective optimization problem.

There are many methods to solve the multi-objectives optimization problems. One basic method is to transfer the multi-objective optimization problems into a single-objective optimization problem. We can divide these methods into two different types. The first alternative is to select one important objective function as the objective function to optimize while the rest of objective functions are defined as constrained conditions. The second alternative is to construct only one evaluation function for optimization by weighting the multiple objective functions. The first method is defined by Markowitz mean–variance model (Markowitz, 1952).

2.1. Type 1: Markowitz mean–variance model

The first method is defined by Markowitz mean–variance model. In Markowitz mean–variance model, the security selection of risky portfolio construction is considered as one objective function and the mean return is defined as one of the constraints. This model is described as:

$$\mathrm{Min}\sum_{i=1}^{N}\sum_{j=1}^{N}w_i w_j \sigma_{ij} \tag{1}$$

$$\text{Subject to } \sum_{i=1}^{N}w_i r_i = \mathrm{R}^*, \tag{2}$$

$$\sum_{i=1}^{N}w_i = 1, \tag{3}$$

$$0 \leqslant w_i \leqslant 1 \quad i = 1,\ldots,N, \tag{4}$$

where N is the number of different assets, σ_{ij} is the covariance between returns of assets i and j, w_i is the weight of each stock in the portfolio, r_i is the mean return of stock i and R^* is the desired mean return of the portfolio.

2.2. Type 2: single objective function model

The second method is to construct only one evaluation function for modeling a portfolio optimization problem. Efficient Frontier and Sharpe Ratio models are described as the following:

2.2.1. Efficient Frontier

We can find the different objective function values by varying desired mean return R^*. The standard practice introduces a new risk aversion parameter $\lambda \in [0,1]$. With this new parameter λ, the model can be described as one objective function:

$$\mathrm{Min}\lambda\left[\sum_{i=1}^{N}\sum_{j=1}^{N}w_i w_j \sigma_{ij}\right] - (1-\lambda)\left[\sum_{i=1}^{N}w_i r_i\right] \tag{5}$$

$$\text{Subject to}\sum_{i=1}^{N}w_i = 1, \tag{6}$$

$$0 \leqslant w_i \leqslant 1 i = 1,\ldots,N. \tag{7}$$

When λ is zero, the model maximizes the mean return of the portfolio, regardless of the variance (risk). In contrast, when λ equals unity, the model minimizes the risk of the portfolio regardless of the mean return. So the sensitivity of the investor to the risk increases as λ increasing from zero to unity, while it decreases as λ approaches zero.

Each case with different λ value would have a different objective function value, which is composed of mean value and variance (risk). Tracing the mean return and variance intersections with different parameter λ, we can draw a continuous curve that is called an efficient frontier in the Markowitz theory (Markowitz, 1952). Since each point on an efficient frontier curve indicates an optimum, and this indicates the portfolio optimization problem is a multi-objective optimization problem. The introducing parameter λ makes the problem to be transfer into a single-objective function problem.

2.2.2. Sharpe Ratio model

Instead of focusing on the mean variance efficient frontier, we seek to optimize the portfolio Sharpe Ratio (SR) (Sharpe, 1966). The Sharpe Ratio combines the information from mean and variance of an asset. It is quite simple and it is a risk-adjusted measure of mean return, which is often used to evaluate the performance of a portfolio. It is described with the following equation:

$$SR = \frac{R_p - R_f}{StdDev(p)}, \tag{8}$$

where p is the portfolio, R_p is the mean return of the portfolio p, R_f is the test available rate of return of a risk-free security (i.e. the interest rate on a three-month U.S. Treasury bill). StdDev(p) is the standard deviation of R_p, in other words, it is a measure of risk of the portfolio. Adjusting the portfolio weights w_i, we can maximize the portfolio Sharpe Ratio in effect balancing the trade-off between maximizing the expected return and at the same time minimizing the risk. In this study, Sharpe Ratio is used in the PSO in order to find the most valuable portfolio with good stock combinations.