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Using homogeneous groupings in portfolio management

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ABSTRACT

Often, in situations of uncertainty in portfolio management, it is difficult to apply the numerical methods based on the linearity principle. When this happens it is possible to use nonnumeric techniques to assess the situations with a nonlinear attitude. One of the concepts that can be used in these situations is the concept of grouping. In the last 30 years, several studies have tried to give good solutions to the problems of homogeneous groupings. For example, we could mention the Pichat algorithm, the affinities algorithms and several studies developed by the authors of this work. In this paper, we use some topological axioms in order to develop an algorithm that is able to reduce the number of elements of the power sets of the related sets by connecting them to the sets that form the topologies. We will apply this algorithm in the grouping of titles listed in the Stock Exchange or in its dual perspective.

1. Introduction

Zadeh (1965) presented the theory of fuzzy sets. Since its creation, fuzzy set theory has been extensively analyzed by many authors and applied to practically all the known sciences. One of the first disciplines where fuzzy set theory was implemented was mathematics. And inside mathematical sciences, one of the first branches that were studied with fuzzy set theory was topology (Chang, 1968). Since its introduction, fuzzy topology has been studied by a wide range of authors (Bayoumi, 2005; Fang & Chen, 2007; Fang & Yue, 2004; Gil-Aluja, 1999; Gil-Aluja & Gil-Lafuente, 2007; Höhle & Rodabaugh, 1999; Hutton, 1975, 1977; Lowen, 1976, 1977; Rodabaugh & Klement, 2003; Saadati & Park, 2006; Solovyov, 2009; Yan & Wu, 2007; Yang & Zhang, 2009; Yue, 2007). Note that a more general context to this approach can be considered when using fuzzy pretopological structures (Badard, 1981, 1984, 1999; Gil-Aluja, 2003; Gil-Aluja & Gil-Lafuente, 2007; Kaufmann, 1983).

The objective of this paper is to present a new framework in portfolio management that it is based on the use of some topological axioms. We present an algorithm that it is able to reduce the number of elements of the power sets of the related sets by connecting them to the sets that form the topologies. We also develop an illustrative example in the grouping of titles listed in the Stock Exchange or in its dual perspective.

Note that the research presented in this paper is mainly based on the previous research carried out by Gil-Aluja (1999, 2003), Gil-Aluja and Gil-Lafuente (2007), Gil-Aluja, Gil-Lafuente, and Klimova (2009) consisting in applying fuzzy topological and pretopological theories in business applications. As far as we know, fuzzy set theory has been implemented in a wide range of business applications (Canós & Liern, 2008; Chen, Huang, & Lin, 2009; Gil-Lafuente, 2005; Gil-Lafuente & Merigó, 2010; Kaufmann, 1975; Kaufmann & Gil-Aluja, 1986; Li & Ho, 2009; Lin, Lin, Hsiao, & Lin, 2009; Merigó & Casanovas, 2011; Merigó & Gil-Lafuente, 2008, 2009, 2010; Merigó, Gil-Lafuente, & Barcellos, 2010). However, the use of fuzzy topology in business problems is very uncommon in the literature. Therefore, we believe that this paper can be very useful for analyzing business problems from a topological context.

The rest of the paper is organized as follows. In Section 2 we present the two basic perspectives for topological fuzzification in economy. Section 3 analyzes the hypothesis of a referential set of referentials. Section 4 presents the dual approach and Section 5 analyzes the relation between the two topologies. In Section 6 we introduce an algorithm for the maximum relations between the titles and the attributes and in Section 7 we end the paper summarizing the main conclusions.

2. The two perspectives for topological fuzzification in economy

It is well known that a topology E in uncertainty can be defined by the subset $T(E)$ of the opened that accomplishes the following axioms (Chang, 1968). Note that for further reading on fuzzy topology and pretopology, we recommend, for example (Badard, 1981;

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Bayoumi, 2005; Fang & Chen, 2007; Fang & Yue, 2004; Gil-Aluja, 2003; Gil-Aluja & Gil-Lafuente, 2007; Saadati & Park, 2006; Yan & Wu, 2007; Yue, 2007):

1. $\emptyset \in T(E)$.
2. $E \in T(E)$.
3. $(A_j \in T(E), A_k \in T(E)) \rightarrow (A_j \cap A_k \in T(E))$.
4. $(A_j \in T(E), A_k \in T(E)) \rightarrow (A_j \bigcup A_k \in T(E))$.

where A_j and A_k may have a different meaning depending on the criteria used for the fuzzification. In the first case, they are fuzzy subsets of the referential set E that accomplishes the previous axioms, and in the other case, they are elements of the power set established from a referential set E of fuzzy subsets. In the first case, the referential set E is formed initially by the elements of the referential set of the fuzzy subsets. In the second case, their elements are the fuzzy subsets theirselves. As it has been pointed out in other works (Gil-Aluja, 2003; Gil-Aluja & Gil-Lafuente, 2007) the selection of one of these perspectives depends mainly on the objectives of the analysis.

In an economical and financial context, we consider that it is relevant to think about the meaning of the components of both cases. For doing this, we will use the representability of the notion of fuzzy subset. The reason is because for an economist, a fuzzy subset is a descriptor of a physical or mental object; and this description is developed by putting different levels to the elements of the referential set formed by the attributes of the objects that we want to describe. Then, in the economic environment it is possible to accept that in the first case, the referential set E is formed by the set of attributes that describe each object while in the second case, the referential set E is formed by the fuzzy subsets, where each of them describe an object.

If we consider financial products such as titles listed in the Stock Exchange, the description of each of them will take place by a certain number of attributes such as the expected rentability, the liquidity capacity without loses, etc., all of them classified at certain level. In this assumption, the referential set E will be formed in the first case by the expected rentability, the liquidity capacity, etc., and in the second case, by the different titles listed in the Stock Exchange.

With this approach, the A_j and $A_k \in \mathrm{T}(E)$, the elements of the open set $T(E)$, are in the first case, fuzzy subsets with the referential of their attributes and in the second case, fuzzy subsets or groupings of fuzzy subsets with the same referential.

It is obvious that the concept of economic representability is different in each case. Thus, the axioms 1 and 2 obtain the following meaning:

- In the first case, axiom 1 shows that the fuzzy subset (title listed in the Stock Exchange) with a null level in all its attributes is an open set and so is (axiom 2) the fuzzy subset (title listed in the Stock Exchange) with level one (maximum) in all its attributes.
- In the second case, axiom 1 shows that in a situation without fuzzy subsets we have an open set. In this case, the set of all the fuzzy sets (all the titles listed in the Stock Exchange) is also an open set (axiom 2).

In axiom 3, we also find different meanings depending on the case analyzed:

- In the first one, axiom 3 requires that if a fuzzy subset with certain levels for each attribute is an open set and so is another fuzzy subset with its own levels, then, there exists a third one that it is also an open set with a membership level for each attribute that is equal to the lowest of the other two.
- In the second one, we can see that if a group of fuzzy subsets is an open set and so is another group of fuzzy subsets, then, the group of fuzzy subsets that is contained in both groups, is also an open set.

Finally, axiom 4 expresses the following for each case:

- In the first one, if we have a fuzzy subset with certain levels for each attribute and another one with its own levels, and both are open sets, then, there exists another fuzzy subset that it is also an open set. The membership level of the attributes of this fuzzy subset is given by the maximum between the other two fuzzy subsets.
- In the second one, if we have two groups of fuzzy subsets that are open sets, then, there exists a third one that is also an open set and it comprises the fuzzy subsets of the first and/or the second group.

Sometimes, it can be useful to use as open sets, the complementary of any open set. This implies the necessity of considering another axiom as follows:

5. $(A_j \in T(E)) \rightarrow (\overline{A}_j \in T(E))$.

In this axiom, the representativity also acquires a different meaning depending on the case used. Then:

- In the first case, it is necessary that if a fuzzy set is an open set, then, the fuzzy subset which has a complimentary level to the first one in all the attributes has also to be an open set. Then, if for a certain attribute an open set has a level α the complimentary fuzzy subset will have $1-\alpha$, where $\alpha \in [0,1]$.
- In the second case, when a grouping of fuzzy subsets is an open set, then, the group formed by the rest of fuzzy subsets is also an open set.

Focusing in this important context, we believe that it is interesting to note that it is not necessary to establish the existence of the five axioms presented above for arriving to the same result. This happens because if three of the axioms are accomplished, then, the other two will be accomplished automatically. These three axioms are:

(1) $E \in \mathrm{T}(E)$.
(2) $(A_j \in T(E)) \rightarrow (A_j \in \mathrm{T}(E))$.
(3) $(A_j \in \mathrm{T}(E), A_k \in \mathrm{T}(E)) \rightarrow (A_j \bigcup A_k \in T(E))$.

As we can see, with the first and the second axiom, it is satisfied:

$$\emptyset \in T(E).$$

And, due to:

$$A_j \cup A_k \in T(E).$$

Is also:

$$\overline{A_j \cup A_k} \in T(E).$$

By using De Morgan theorem:

$$\overline{A_j \cup A_k} = \overline{A}_j \cap \overline{A}_k.$$

Then:

$$\overline{A}_j \cap \overline{A}_k \in T(E).$$