CIRP Annals - Manufacturing Technology 57 (2008) 463–466

Contents lists available at ScienceDirect

CIRP Annals - Manufacturing Technology

journal homepage: http://ees.elsevier.com/cirp/default.asp

Dynamic modeling of production networks of autonomous work systems with local capacity control

N.A. Duffie (1)[a,*], D. Roy[b], L. Shi[b]

[a] *Department of Mechanical Engineering, University of Wisconsin-Madison, Madison, WI, USA*
[b] *Department of Industrial and Systems Engineering, University of Wisconsin-Madison, Madison, WI, USA*

ARTICLE INFO

Keywords:
Production
Control
Dynamic model

ABSTRACT

In this paper, a dynamic model is presented for production networks with a potentially large number of autonomous work systems, each having local capacity control. The model allows fundamental dynamic properties to be predicted using control-theoretic methods, together with the response of variables such as work-in-progress and lead-time for the network and its individual work systems. This is illustrated using industrial data. The behavior of one of the work systems in this network is analyzed further, and the results are compared with results obtained using a discrete event simulation model.

1. Introduction

Production networks are emerging as a new type of cooperation between and within companies, requiring new techniques and methods for their operation and management [1]. Coordination of resource use is a key challenge in achieving short delivery times and delivery time reliability. These networks can exhibit unfavorable dynamic behavior as individual organizations respond to variations in orders in the absence of sufficient communication and collaboration, leading to recommendations that supply chains should be globally rather than locally controlled and that information sharing should be extensive [2,3]. However, the dynamic and structural complexity of these emerging networks inhibits collection of the information necessary for centralized planning and control, and decentralized coordination must be provided by logistic processes with autonomous capabilities [4]. Dynamic models will be an important tool in understanding the behavior of networks with decentralized control and enabling design of effective autonomous logistic processes.

A production network with several autonomous work systems is depicted in Fig. 1. The behavior of such a network is affected by external and internal order flows, planning, internal disturbances, and the control laws used locally in the work systems to adjust resources for processing orders. In prior work, sharing of capacity information between work systems has been modeled [5] along with the benefits of alternative control laws and reducing delay in capacity changes [6]. Several authors have described both linear and nonlinear dynamical models for control of variables such as inventory levels and work in progress (WIP), including the use of pipeline flow concepts to represent lead times and production delays [7]. A closed-loop production planning and control concept has been employed with adaptive inventory control in decision support systems in a multi-product medical supplies market [8]. State-space models have been used for switching between a library of optimal controllers to adjust WIP in serial production systems in the presence of machine failures [9], and switching of control policies in response to market strategies has been investigated [10]. In this paper, the focus is on development of a discrete state-space dynamic model for production networks with an arbitrarily large number of work systems, illustrating the use of this generic model to predict performance, and comparing the results with results obtained using discrete event simulation.

2. Model of multiple work systems

In Fig. 1, the elements of vector $\mathbf{i}(nT)$ represent the rate at which orders flow from external sources into N work systems in a network at discrete instants in time separated by time interval T (e.g. 1 shop calendar day [scd]),where $n = 0, 1, 2, \ldots$. The total number of orders that have been input to each work system then can be represented by the elements of vector

$$\mathbf{w}_{\mathrm{i}}((n+1)T) = \mathbf{w}_{\mathrm{i}}(nT) + T(\mathbf{i}(nT) + \mathbf{P}^{T}\mathbf{c}_{\mathrm{a}}(nT)) \tag{1}$$

where $\mathbf{c}_{\mathrm{a}}(nT)$ is the actual capacity (rate at which orders are output) of each work system and $\mathbf{P}$ is a matrix in which element p_{jk} approximates the fraction of the flow out of work system j that flows into work system k [11]. The total number of orders that have been output by each work system is

$$\mathbf{w}_{\mathrm{o}}((n+1)T) = \mathbf{w}_{\mathrm{o}}(nT) + T\mathbf{c}_{\mathrm{a}}(nT) \tag{2}$$

and the WIP is

$$\mathbf{wip}_{\mathrm{a}}(nT) = \mathbf{w}_{\mathrm{i}}(nT) - \mathbf{w}_{\mathrm{o}}(nT) + \mathbf{w}_{\mathrm{d}}(nT) \tag{3}$$

where $\mathbf{w}_{\mathrm{d}}(nT)$ represents local work disturbances, such as rush orders, that affect the work systems.

If it is desired to maintain the WIP in each work system in the vicinity of planned levels $\mathbf{wip}_{\mathrm{p}}(nT)$, then the capacities of the work

* Corresponding author.

0007-8506/$ – see front matter © 2008 CIRP.
doi:10.1016/j.cirp.2008.03.018

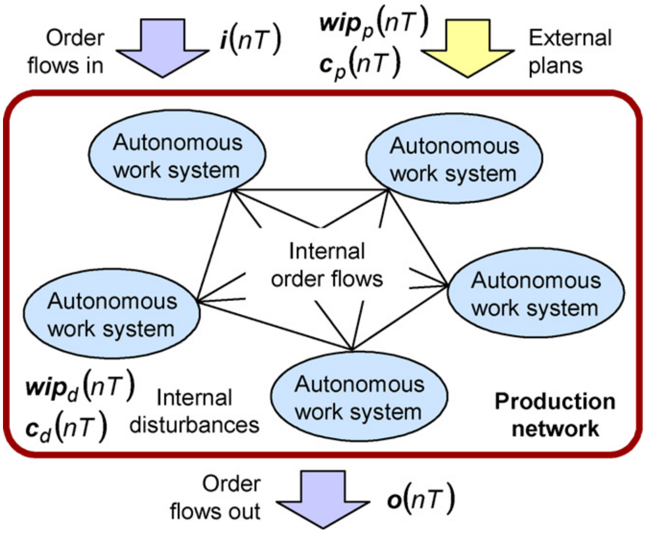

Fig. 1. Production network consisting of a group of autonomous work systems.

systems can be adjusted with respect to their planned capacities $\mathbf{c}_\mathrm{p}(nT)$ using straightforward proportionality k_c:

$$\mathbf{c}_\mathrm{m}(nT) = k_\mathrm{c}(\mathbf{wip}_\mathrm{a}(nT) - \mathbf{wip}_\mathrm{p}(nT)) \tag{4}$$

$$\mathbf{c}_\mathrm{a}(nT) = \mathbf{c}_\mathrm{p}(nT) + \mathbf{c}_\mathrm{m}((n-d)T) - \mathbf{c}_\mathrm{d}(nT) \tag{5}$$

where $\mathbf{c}_\mathrm{d}(nT)$ represents local capacity disturbances such as equipment failures. The value of k_c determines how quickly differences between actual and planned WIP are eliminated, and the whether work system responses are fundamentally oscillatory (regardless of whether there are oscillations in their inputs). Furthermore, capacity changes were assumed to be delayed by a time period dT, but planned capacity and WIP were assumed to be known in advance and did not require delayed implementation. Although factors such as setup times and WIP are known to affect performance [12], it was assumed that the actual capacity of the work systems was equal to their full capacity of the work systems. There was no information sharing between work systems.

Eqs. (1) through (5) can be combined and transformed to obtain a discrete model for the system:

$$\begin{bmatrix} z\mathbf{w}_\mathrm{i}(z) \\ z\mathbf{w}_\mathrm{o}(z) \\ z^d\mathbf{c}_\mathrm{m}(z) \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{0} & T\mathbf{P}^T \\ \mathbf{0} & \mathbf{I} & T\mathbf{I} \\ k_\mathrm{c}\mathbf{I} & -k_\mathrm{c}\mathbf{I} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{w}_\mathrm{i}(z) \\ \mathbf{w}_\mathrm{o}(z) \\ \mathbf{c}_\mathrm{m}(z) \end{bmatrix} + \begin{bmatrix} T\mathbf{I} & \mathbf{0} & \mathbf{0} & T\mathbf{P}^T & -T\mathbf{P}^T \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & T\mathbf{I} & -T\mathbf{I} \\ \mathbf{0} & k_\mathrm{c}\mathbf{I} & -k_\mathrm{c}\mathbf{I} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{i}(z) \\ \mathbf{w}_\mathrm{d}(z) \\ \mathbf{wip}_\mathrm{p}(z) \\ \mathbf{c}_\mathrm{p}(z) \\ \mathbf{c}_\mathrm{d}(z) \end{bmatrix} \tag{6}$$

$$\begin{bmatrix} \mathbf{w}_\mathrm{i}(z) \\ \mathbf{w}_\mathrm{o}(z) \\ \mathbf{c}_\mathrm{a}(z) \\ \mathbf{wip}_\mathrm{a}(z) \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \\ \mathbf{I} & -\mathbf{I} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{w}_\mathrm{i}(z) \\ \mathbf{w}_\mathrm{o}(z) \\ \mathbf{c}_\mathrm{m}(z) \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} & -\mathbf{I} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{i}(z) \\ \mathbf{w}_\mathrm{d}(z) \\ \mathbf{wip}_\mathrm{p}(z) \\ \mathbf{c}_\mathrm{p}(z) \\ \mathbf{c}_\mathrm{d}(z) \end{bmatrix} \tag{7}$$

It is desirable to obtain a discrete state-space-model that is compatible with dynamic systems analysis software (Matlab was used to obtain the results in this paper); therefore, if $d = 0$ then the state $\mathbf{c}_\mathrm{m}(z)$ should be eliminated in Eq. (6), whereas Eq. (6) should be augmented if $d > 1$ so that there are d states $\mathbf{c}_\mathrm{m}(z), \mathbf{c}'_\mathrm{m}(z), \mathbf{c}''_\mathrm{m}(z)$, etc., each representing a delay of T in capacity adjustment. As a result, there will be $N(d+2)$ states, $5N$ inputs and $4N$ outputs in the dynamic model for the production network. Hence, its dimensions grow linearly with the number of work systems and linearly with the delay in capacity adjustment. Two outputs can be added to Eq. (7), if desired, that represent the total orders input to the network and output from the network:

$$w_\mathrm{in}(nT) = [1 \quad 1 \quad \ldots \quad 1](\mathbf{w}_\mathrm{i}(nT) - \mathbf{P}^T\mathbf{w}_\mathrm{o}(nT)) \tag{8}$$

$$w_\mathrm{out}(nT) = [1 \quad 1 \quad \ldots \quad 1]\mathbf{P}_0\mathbf{w}_\mathrm{o}(nT) \tag{9}$$

where $\mathbf{P}_0$ is the diagonal matrix with non-zero elements

$$p_{0_{jj}} = 1 - \sum_{k=1}^{N} p_{jk} \tag{10}$$

Furthermore, the rate at which orders flow out of the network from the individual work systems is

$$\mathbf{o}(nT) = \mathbf{P}_0\mathbf{c}_\mathrm{a}(nT) \tag{11}$$

3. Multiple work system example

Application of this discrete state-space dynamic model was illustrated using data from a forging company that supplies components to the automotive industry. The company's basic product is starter ring gears; other products include sensor wheels with machined teeth and flywheel assemblies for manual transmissions. The data documents 659 orders that entered the system from scd 162 to scd 347 in the year 2001. For purposes of analysis, the production system was grouped into the five work systems listed in Table 1. Eqs. (6) and (7) accommodate planned capacities and WIP that vary with time. The planned capacities and WIP listed in Table 1 they are averages obtained from the data. The time periods over which the plan was used in the model are also listed in Table 1. The planned capacities and WIP were zero 14 days before and after these periods; however, Eqs. (4) and (5) allow output rates and WIP to be greater than zero at such times. The internal flow of orders was approximated using the following matrix, in which element p_{ij} is the total number of orders that went from work system i to work system j divided by the total number of orders that left work system i:

$$\mathbf{P} = \begin{bmatrix} 0 & 106/341 & 235/341 & 0 & 0 \\ 0 & 0 & 0 & 188/401 & 204/401 \\ 0 & 0 & 0 & 100/236 & 129/236 \\ 0 & 0 & 0 & 0 & 268/295 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{12}$$

Fig. 2 shows the order flows into each work system in the network. With these inputs, the plan in Table 1, the order flow matrix in Eq. (12), and zero initial conditions, the dynamic mode with $k_c = 0.25$ scd^{-1} and $T = 1$ scd predicts the throughput shown in Fig. 3.

A transfer function can be obtained for each output/input combination in Eq. (7); for example, the change in actual capacity that results from capacity disturbances in the Shearing/Sawing

Table 1
Planned capacity and WIP

Work system	c_p (orders/scd)			$\mathbf{wip}_p$ (orders)	Duration[a] (scd)
	Monday–Friday	Saturday	Sunday		
1. Shear/Sawing	4.72	0.92	0.00	21.07	181–244
2. Ring rolling	5.34	1.50	0.00	18.92	181–244
3. Drop forging	2.95	0.42	0.00	14.46	181–244
4. Heat treat.	2.70	2.50	1.92	14.87	181–244
5. Quality control	6.28	0.83	0.08	72.11	188–265

[a] Preceded/followed by a 14-day linear ramp up/down period.