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Optimization of imperfect preventive maintenance for multi-state systems

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Abstract

The paper generalizes a preventive maintenance optimization problem to multi-state systems, which have a range of performance levels. Multi-state system reliability is defined as the ability to satisfy given demand. The reliability of system elements is characterized by their hazard functions. The possible preventive maintenance actions are characterized by their ability to affect the effective age of equipment. An algorithm is developed which obtains the sequence of maintenance actions providing system functioning with the desired level of reliability during its lifetime by minimum maintenance cost.

To evaluate multi-state system reliability, a universal generating function technique is applied. A genetic algorithm (GA) is used as an optimization technique. Basic GA procedures adapted to the given problem are presented. Examples of the determination of optimal preventive maintenance plans are demonstrated. © 2000 Elsevier Science Ltd. All rights reserved.

Keywords: Imperfect preventive maintenance; Universal generating function; Genetic algorithm

1. Introduction

The evolution of system reliability depends on its structure as well as on the evolution of the reliability of its elements. The latter is a function of element age on a system's operating life. Element aging is strongly affected by maintenance activities performed on the system. Although in some special cases surveillance or maintenance can produce an increment in the effective age of the equipment [1], in this paper we consider the maintenance actions that are characterized by their ability to reduce this age.

Preventive maintenance consists of actions, which improve the condition of system elements before they fail. PM actions such as the replacement of an element by a new one, cleaning, adjustment, etc. either return the element to its initial condition (the element becomes "as good as new") or reduce the age of the element. In some cases the PM activity (surveillance) does not affect the state of the element but ensures that the element is in operating condition. In this case the element remains "as bad as old". All actions that do not reduce to zero element age can be considered to be imperfect PM. When an element of the system fails, corrective maintenance in the form of minimal repair is performed which returns the element to operating condition without affecting its failure rate.

Optimizing the policy of preliminarily planned PM actions with minimal repair at failure for systems with increasing element failure rates is the subject of much research [2–9]. All of these works consider binary-state systems reliability.

When applied to multi-state systems, reliability is considered to be a measure of the ability of a system to meet demand (required performance level). For example, in power engineering, the ability of a system to provide an adequate supply of electrical energy [10,11] is used for evaluating its availability. In this case, the outage effect will be essentially different for units with different nominal capacity and will also depend on consumer demand. Therefore, the performance rates (productivity) of system elements should be taken into account as well as the level of demand when the entire system's reliability is estimated.

The general definition of MSS reliability according to Ref. [12] is:

$$R_{\mathrm{MSS}}(t, W) = \Pr\{G_{\mathrm{MSS}}(t) \geq W\}, \tag{1}$$

where $G_{\mathrm{MSS}}(t)$ is output performance of the MSS at time t and W is required MSS output performance (demand).

For MSS which have a finite number of states there can be K different levels of output performance at each time t: $G(t) \in \mathbf{G} = \{G_k, 1 \leq k \leq K\}$ and system OPD can be defined by two finite vectors $\mathbf{G}$ and $\mathbf{q} = \{q_k(t)\} = \Pr\{G(t) = G_k\}$ $(1 \leq k \leq K)$. Therefore MSS reliability is the probability that a system remains in those states in

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Nomenclature

T	MSS lifetime
$\tau_j(\mathrm{t})$	effective age of the MSS element j at chronological time t
$\tau_j^+(t)$	effective age of the MSS element j immediately after performing a PM action at chronological time t
t_{ji}	chronological time of performing ith PM on element j
ϵ	age reduction coefficient
$r_j(t)$	reliability of jth element
$h_j(t)$	hazard function of jth element
$H_j(t)$	accumulated hazard function for jth element
n_j	number of PM actions performed on MSS element j during lifetime T
N	total number of PM actions performed on MSS during T
J	total number of elements composing MSS
G_j	nominal performance rate of the jth MSS element
G_{MSS}	output performance rate of the entire MSS
W	system demand
$R_{\mathrm{MSS}}(t,W)$	probability that $G_{\mathrm{MSS}} \geq W$ at time t (MSS reliability)
R^*	desired MSS reliability
v_{i}	number (type) of ith PM action to be performed
$\mathbf{V}$	vector of PM actions (PM plan)
c_j	cost of minimal repair of element j
$C_{\mathrm{M}j}$	cost of minimal repairs of element j in interval $[0,t]$
$C_{\mathrm{P}}(v)$	cost of PM action number v
$C_{\mathrm{P}}(\mathbf{V})$	total cost of PM actions
$C_{\mathrm{M}}(\mathbf{V})$	total cost of minimal repairs
$C_{\mathrm{tot}}(\mathbf{V})$	total MSS maintenance cost
θ	time interval between possible PM actions
K	number of MSS states corresponding to different output performance levels
N_{S}	number of randomly constructed solutions in initial population of GA
N_{rep}	number of reproduction procedures during genetic cycle of GA
N_{c}	number of cycles in GA

Acronyms[1]

CM	corrective maintenance
GA	genetic algorithm
MSS	multi-state system
PM	preventive maintenance
OPD	output performance distribution
UMGF	universal moment generating function

[1] The singular and plural forms of acronyms are always spelled the same.

which $G_{\mathrm{k}} \geq W$ during $(0,t)$:

$$R_{\mathrm{MSS}}(t,W) = \sum_{G_k \geq W} q_k(t). \qquad (2)$$

A method for evaluating the reliability of series–parallel MSS consisting of elements with different performance rates was suggested in [13]. This method, based on universal generating functions, proved to be convenient for numeric implementation and effective at solving problems of MSS redundancy and maintenance optimization [14,16], as well as importance analysis [17]. The method can also be used for evaluating the influence of PM actions applied to specific elements on entire MSS reliability. Unlike fault-tree analysis, the universal generating function method provides for the possibility of treating systems with similar topologies but with different nature of elements interaction in a similar way.

In this paper we present an algorithm which determines a minimal cost plan of PM actions during MSS lifetime, which provides the required level of system reliability. The algorithm answers the questions of when, where (to which element) and what kind of available PM actions should be applied to keep the system on the required level of output performance with desired reliability during a specified time.

To solve the problem, a genetic algorithm is used. The solution encoding technique is adapted to represent replacement policies. A solution quality index comprises both reliability and cost estimations.

An illustrative example is presented in which the optimal PM plan is found for a series–parallel system.

2. Imperfect PM model

Imperfect PM is modeled using the age reduction concept [18]. According to this concept the PM action reduces the effective age of the element that it has immediately before it enters maintenance. The used proportional age setback model [19] assumes that the effective age τ_j of element j which undergoes PM actions at chronological times

$$(t_{j1}, \ldots, t_{jn}) \qquad (3)$$

is

$$\tau_j(t) = \tau_j^+(t_{ji}) + (t - t_{ji}) \qquad \text{for } t_{ji} < t < t_{ji+1}\ (0 \leq i \leq n), \qquad (4)$$

$$\tau_j^+(t_{ji}) = \epsilon_i \tau_j(t_{ji}) = \epsilon_i[\tau_j^+(t_{ji-1}) + (t_{ji} - t_{ji-1})],$$

where $\tau_j^+(t_{ji})$ is the age of the element immediately after the ith PM, ϵ_i is the age reduction coefficient associated with the ith PM action which ranges in the interval [0,1], and $\tau_j(0) = t_{j0} = 0$ by definition.

The two extreme effects of PM on the state of the element correspond to the cases when $\epsilon = 1$, or $\epsilon = 0$. In the first case the model simply reduces to “as bad as old” which