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A Bayesian approach to an adaptive preventive maintenance model

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Abstract

In this paper we consider a Bayesian theoretic approach to determine an optimal adaptive preventive maintenance policy with minimal repair. By incorporating minimal repair, maintenance and replacement, the mathematical formulas of the expected cost per unit time are obtained. When the failure density is Weibull with uncertain parameters, a Bayesian approach is established to formally express and update the uncertain parameters for determining an optimal adaptive preventive maintenance policy. Furthermore, various special cases of our model are discussed in detail. © 2001 Elsevier Science Ltd. All rights reserved.

Keywords: Bayesian approach; Minimal repair; Maintenance; Replacement; Reliability; Weibull distribution

1. Introduction

It is important to preventively maintain a system to avoid failures during operation especially when such an event is costly and/or dangerous. The problem that arises in reliability study is when to maintain the system. The number of failures during actual operation should be reduced to as few as possible by means of maintenance. In most maintenance models, it is commonly assumed that a maintenance action (replacement) regenerates the system. For a complex system the maintenance action is not necessarily the replacement of the whole system, but often the repair or replacement of a part of the system. Hence, the maintenance action may not renew the system completely. In this case, Barlow and Hunter [1] considered two types of preventive maintenance policy — one type for single-item systems and another for multi-item systems. These two types of policy have been studied extensively in the literature [2–4]. Furthermore, Nguyen and Murthy [5], Nakagawa [6] and Sheu and Liou [7] investigate a different type of policy for repairable systems, called sequential preventive maintenance policy. In this type of policy, there is a basic assumption that the life distribution of the system changes after each maintenance in such a way that its failure rate function increases with the number of maintenance actions.

In practice, replacement policies are prevailingly adopted to preventively maintain a system such as the age replacement policy and the block (periodic) replacement policy. In addition to replacement action, Barlow and Hunter [1] generalize the replacement policy by incorporating minimal repairs at failures. This model has been intensively investigated for various cases when the failure distribution of the system is known with certainty [8–15]. However, the failure distribution of a system is usually unknown or known with uncertain parameters in practice. In this case, it is necessary to select an appropriate estimation method to calculate accurately the parameter(s) of a given distribution and the expected mean life of the system. Researchers in this field include Gibbons and Vance [16], Lawless [17], Mann [18] Pan and Chen [19], Sinha and Sloan [20], Soland [21], Thoman et al. [22] and Varde [23]. In particular, Sathe and Hancock [24] adopt a Bayesian approach by considering prior distributions on the shape and scale parameters of a Weibull failure distribution to derive the optimal replacement policy such that the expected long-run average cost is minimized.

Taking a further step, Willson and Benmerzouga [25] investigate Bayesian group replacement policies for the case that the failure times are exponentially distributed. Bassin [26] introduce a Bayesian block replacement policy for a Weibull restoration process and derive the optimal overhaul interval when the expected repair cost is known. Moreover, when the repair cost is constant, Mazzuchi and Soyer [27] employ the Bayesian decision theoretic approach and develop a Weibull model for both the block replacement protocol with minimal repair and the traditional age replacement protocol. However, the repair cost for system failures may be random and unknown in practice. In this paper,

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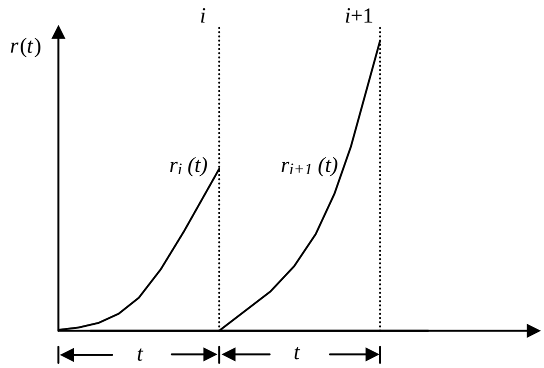

Fig. 1. The failure functions of system after maintenance.

we extend Mazzuchi and Soyer's model by allowing the minimal repair cost to be random. Furthermore, we propose an adaptive preventive maintenance model for a repairable system and develop a Bayesian technique to derive the optimal maintenance policy.

In our model, a planned maintenance is carried out as soon as T time units have elapsed since the last maintenance action, if at Nth maintenance, the system is replaced rather than maintained. Furthermore, when the system fails before age T, it is either correctively maintained (or replaced after $(N-1)$ maintenances) or minimally repaired depending on the random repair cost at failure. Here maintenance means planned maintenance or unplanned maintenance (corrective maintenance). The objective is to determine the optimal plan (in terms of N and T) which minimizes expected cost per unit of time. When the failure density is Weibull, a Bayesian approach is proposed to derive the optimal adaptive preventive maintenance policy with minimal repair such that the expected cost per unit time is minimized.

The remainder of this paper is organized as follows. In the second section, the extended adaptive preventive maintenance policy is described in detail and the expected cost per unit time is formulated. In the third section, a Bayesian decision theoretic approach is established when the failure density is Weibull. Finally, some special cases of our model are discussed in detail.

2. Adaptive preventive maintenance model

Suppose that maintenance actions include planned actions if the system does not fail and corrective actions if the system fails before age T. After each maintenance, the system is operational but its failure characteristic is altered. More precisely, let $r_i(t)$ denote the failure rate at time t (t hours from the last maintenance) of a system subjected to $(i-1)$ maintenance actions. The basic assumptions are as follows:

A1. $r_i(t)$ is strictly increasing and $r_i(t) \to \infty$ as $t \to \infty$.
A2. $r_{i+1}(t) \geq r_i(t)$, for any $t > 0$.
A3. $r_{i+1}(0) = r_i(0)$, for all i. The case $i = 1$ corresponds to a new system.

Therefore, the failure rate functions of a system subjected to i and $i+1$ maintenances will have the shapes as shown in Fig. 1.

Given the failure rate $r_i(t)$, the failure distribution function $F_i(t)$, the survival function $\bar{F}_i(t) = 1 - F_i(t)$ or the probability density function $f_i(t)$ can be easily obtained. Furthermore, it is assumed that the time required for performing replacement, maintenance and minimal repair actions are negligible.

For the system mentioned above, the adaptive preventive maintenance model is described as follows. A planned maintenance with cost C_P is performed when the age of the system is T and planned replacement with cost C_R at the Nth maintenance ($C_R > C_P$). If the system fails before T, it is either correctively maintained (or replaced after $(N-1)$ maintenances) or minimally repaired depending on the random repair cost C, which follows a distribution H with density h. Each failure of the system incurs a breakdown with cost C_B. Then, $(C_P + C_B)$ and $(C_R + C_B)$ are the unplanned maintenance and unplanned replacement costs at failure, respectively. When a failure with repair cost $C \leq \delta(C_P + C_B)$ occurs where $0 \leq \delta \leq 1$, it is called a type I failure (minor failure) and a minimal repair is performed. On the contrary, if $C > \delta(C_P + C_B)$, then the failure is classified as a type II failure (catastrophic failure) and a corrective maintenance takes place. That is, a maintenance action is performed at age T or at first type II failure, whichever occurs first. The system is replaced after $N-1$ maintenance actions and the procedure is repeated after a replacement. We also assume all failures are instantly detected.

Let q and p be the probabilities that a failure is classified as a type I and type II failure, respectively. It is clear that $q = P(C \leq \delta[C_P + C_B])$ and $p = 1 - q$. Since a minimal repair is performed only when $C \leq \delta(C_P + C_B)$,the minimal repair cost $\tilde{C}$ incurred follows the density $h_q(x) = h(x)/P(C \leq \delta[C_P + C_B]) = h(x)/q$ for $0 \leq x \leq \delta(C_P + C_B)$, and its expected value is $E[\tilde{C}] = C^*/q$ where $C^* = \int_0^{\delta(C_P + C_B)} x h(x)\,dx$.

Consider the failure process of the system for the case when minimal repair is performed for both type I and II failures. Let $N_i(t)$ be the number of failures in $[0,t]$ during the ith maintenance. Then, the process $\{N_i(t),\ t \geq 0\}$ is a non-homogeneous Poisson process with intensity function $r_i(t)$ [28]. As shown in Savits [29], $\{N_i(t),\ t \geq 0\}$ can be decomposed into two failure processes $\{M_i(t),\ t \geq 0\}$ and $\{L_i(t),\ t \geq 0\}$ for types I and II, respectively. That is, $N_i(t) = M_i(t) + L_i(t)$. It can be easily verified that $\{M_i(t),\ t \geq 0\}$ and $\{L_i(t),\ t \geq 0\}$ are two independent non-homogeneous Poisson processes with intensity functions $qr_i(t)$ and $pr_i(t)$, respectively, [29]. Now, let $Y_i = \inf\{t \geq 0 : L_i(t) = 1\}$ which is the waiting time starting from age 0 until the first type II failure of the system occurs. Then, $M_i(Y_i)$ counts the number of minimal