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Int. J. Production Economics 74 (2001) 147–155

international journal of production economics

www.elsevier.com/locate/dsw

Optimal control of preventive maintenance schedule and safety stocks in an unreliable manufacturing environment

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Abstract

This paper revisits the economic manufacturing quantity (EMQ) model by Cheung and Hausman (Nav. Res. Logist. 44 (1997) 257) from the theoretical point of view, and develops a new stochastic model under somewhat different restrictive assumptions. The optimal policies, the order quantity and safety stock, are derived, respectively, so as to minimize the expected cost per unit time in the steady-state. © 2001 Elsevier Science B.V. All rights reserved.

Keywords: Economic manufacturing; Machine breakdown; Preventive maintenance; Safety stocks; Stochastic model

1. Introduction

When dealing with stocked items, the question regarding how much to order is answered by the economic order quantity (EOQ). Similarly, the economic manufacturing quantity (EMQ) model, which is a natural extension of EOQ, provides the proper production lot size by minimizing the cost components involved, that is, the production cost, the inventory holding cost and the shortage cost if the stockout is permitted. When a machine breakdown takes place in the production phase, however, the basic EMQ model loses its usefulness since the interruption by corrective maintenance operations is aborted. From a practical perspective, the manufacturer should design the production lot from the standpoint of safety, and thus effects of machine breakdown and corrective maintenance in economic production lot sizing decisions should be examined exactly in uncertain environment without reliable manufacturing facilities. In this article, we consider an extended EMQ model with stochastic machine breakdown and repair. To motivate such a modeling, consider the just-in-time (JIT) philosophy. The JIT production whose lot size is required to be small or is ideally equivalent to one, has not been able to be achieved in many manufacturing industries all over the world, since fairly high set-up and machine maintenance costs are needed in practice. In particular, one of the major impediments for the successful operation of such a tightly coupled organization may be formed by breakdowns in bottleneck resources.

Numerous research efforts have been undertaken to extend the manufacturing model subject to stochastic machine breakdowns. Lee and Rosenblatt [1,2] focused on the imperfections in the production process and equipment, and determined the optimal EMQ policy and/or inspection schedule. Furthermore, Rosenblatt and Lee [3] analyzed a deteriorating system during the production

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PII: S0925-5273(01)00121-9

process. Groenevelt et al. [4] and Ibrahim and Kee [5] developed somewhat different stochastic models from the above ones and investigated the effects of machine breakdowns in the optimal lot size and reorder level decisions in the framework of the EMQ model. Also, Dohi and Osaki [6], Dohi et al. [7], Cheung and Hausman [8], Dohi et al. [9,10] considered similar production planning models. Their models are based on a typical maintenance model called *the age replacement model* in Barlow and Proschan [11]. More precisely, Dohi et al. [7] developed an extended EMQ model under the assumption that the lifetime for the production machine obeys a common distribution and derived the optimal lot size which minimizes the expected cost. Cheung and Hausman [8] formulated the optimization problem to determine both the optimal lot size and safety stock, provided that the production rate is equivalent to the demand rate in the normal production phase.

It should be noted that, however, Cheung and Hausman [8] modeled the EMQ process under rather questionable assumptions. This implies that the mathematical treatment is incomplete and at the same time that their model and conclusion cannot be justified. The main purpose of this article is to revisit the economic manufacturing quantity (EMQ) model by Cheung and Hausman [8] from the theoretical point of view, and to develop a new stochastic model under somewhat restrictive assumptions. In order to formulate the expected cost precisely, we assume that the lifetime of the production machine obeys an exponential distribution. This assumption may be more restrictive than the original one by Cheung and Hausman [8], but is required for well-behaved stochastic modelling. This paper is organized as follows. Section 2 describes the EMQ model subject to stochastic machine breakdowns, and introduces the results by Cheung and Hausman [8]. In Section 3, we point out their problems and develop the expected cost function in the improved modelling framework. In Section 4, we derive the optimal lot size and the optimal safety stock minimizing the expected cost, respectively. In addition, the optimization algorithm to seek the optimal lot size and safety stock jointly is developed.

2. EMQ Model with machine breakdown

Let us consider the single-product manufacturing system by Cheung and Hausman [8]. Suppose that the continuous demand for items occurs at rate $p\ (>0)$ per unit time. In the production phase, items are produced at rate p per unit time so as to satisfy the demand. It is noted, however, that the production rate (speed) is controllable and its maximum limit is possibly $p_0\ (>p)$. Also, $s\ (>0)$ safety stocks are always held in the system while the production rate is equivalent to the demand rate.

Let T be a non-negative random variable representing the lifetime of the production machine, with common distribution function $\Pr\{T \leqslant t\} = F(t)$, density $f(t) = \mathrm{d}F(t)/\mathrm{d}t$, failure rate $r_{\mathrm{f}}(t) = f(t)/\bar{F}(t)$ and finite mean $1/\mu\ (>0)$, where in general $\bar{F}(\cdot) = 1 - F(\cdot)$ is the survivor function. It is assumed that $F(t)$ is increasing hazard rate (IHR), that is, $\mathrm{d}r_{\mathrm{f}}(t)/\mathrm{d}t \geqslant 0$. This implies that the production machine is in the initial failure state or the accidental failure state, and tends to fail as time elapses.

In order to avoid the system down by the machine failure in advance, the preventive maintenance is carried out at periodic time interval $m\ (\geqslant 0)$. More specifically, if the production machine does not fail until $t = m$ after the operation starts at time 0, the preventive (major) repair is carried out at that time, where $M\ (0 < M \leqslant s/p)$ time instant (constant) is required to complete the repair and the system becomes as good as new after the repair. From $0 < M \leqslant s/p$, we assume $pM \leqslant s < \infty$. Since the system stops producing items during the preventive repair, the stock level decreases at rate $-p$ uniformly for the time period $[0, M]$. After completing the preventive repair, items are produced at the maximum rate p_0 until the stock level becomes s again, that is, the inventory level increases at rate $p_0 - p$. When the safety stock level is attained, the production rate changes to p.

On the other hand, if the failure occurs before the time m, the corrective (major) repair is placed immediately. Let R denote the non-negative random variable representing the corrective repair time with common distribution function $\Pr\{R \leqslant t\} = G(t)$, density $g(t) = \mathrm{d}G(t)/\mathrm{d}t$, hazard rate $r_{\mathrm{g}}(t) = g(t)/\bar{G}(t)$ and finite mean $1/\lambda(>0)$. During the