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Comput. Methods Appl. Mech. Engrg. 182 (2000) 73–88

Computer methods in applied mechanics and engineering

www.elsevier.com/locate/cma

Shape-stress trade-off design of membrane structures for specified sequence of boundary shapes

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Abstract

A new method is presented for parametrically generating optimal shapes and stresses of a membrane structure supported by a frame. The problem considered here is a so called trade-off design problem where the weighted sum of the deviations of the initial stress and the shape from the desired distribution is to be minimized. The membrane is discretized by using the finite element method, and optimal shapes are expanded with respect to a parameter defining the shape of the boundary frame. An approximate method is presented without exact calculation of the Hessian of the stresses with respect to the nodal coordinates. The explicit developability conditions derived by the authors are used to guarantee the curved surface to be reduced to a set of plane cutting patterns after removing the pretensioning force. In the examples, trade-off designs are found for an HP-type membrane structure and the efficiency of the proposed method is discussed. © 2000 Elsevier Science S.A. All rights reserved.

1. Introduction

In the widely used approach of design and construction of membrane structures, an initial shape of equilibrium is first found for the given boundary conditions and initial stress ratios in two principal directions by using the techniques of form finding analysis [1,2]. Then the curved surface is divided into several cutting patterns. In this process, developability of the curved surface into plane sheets is in the most cases ignored. The initial shape of equilibrium may be defined as a minimal surface [3], or a surface with constant stress that can be found by optimizing a shell structure for membrane state [4]. The equilibrium shape for specified prestress may also be found by using the force density method [1,5]. After the initial shape is determined, the shape of the membrane sheet is usually defined on the basis of geodesic line [6]. The cutting pattern of the membrane without prestresses is calculated by reducing the deformation due to the prestress and the estimated strain corresponding to the relaxation after pretensioning.

The drawbacks of this approach are that the feasible shape is often limited to the surface with constant stress, and the equilibrium shape calculated by connecting and stretching the plane sheets might be far from the specified shape. Another drawback is that the distribution of the stress after pretensioning may not be uniform as expected. The initial shape and stresses may be improved by optimizing the shape of each cutting pattern through an iterative process involving incremental deformation analysis with geometrical non-linearity. In this case, however, substantial computational effort is needed if the number of membrane elements is increased.

Homotopy method or continuation method is an approach for tracing a path of the solutions of a parametrically defined non-linear equations [7–9]. The solutions of a mathematical programming problem which includes a parameter can also be traced by the homotopy method, because an optimization problem

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may be reduced to a problem of finding a solution that satisfies the optimality criteria which is written as a set of non-linear equations if only the active set is incorporated among the inequality constraints. There have been many researches on calculating sensitivity of the solution of a parametric optimization problem with respect to the parameter [10,11]. In the field of structural optimization, the sensitivity of optimal solution with respect to a problem parameter has been discussed in several papers [12–14].

Nakamura and Ohsaki [15] have developed a method of tracing a sequence of optimum designs starting from a trivial solution consisting of the lower bound values of the design variables. The proposed method has been proved to be very effective to the case with multiple fundamental eigenvalues, and has been extended to the design problem under earthquake strain constraints [16]. Nakamura and Ohsaki [17] have also presented a practically efficient method of topology optimization where the lower bound of the design variables are monotonically decreased to null. Ohsaki and Arora [18] applied the parametric programming technique to solving a general constrained optimization problem.

Homotopy method can also be applied to the form finding analysis of tensile structures such as cable nets and membranes. Ohsaki et al. [19,20] presented a method for generating a series of initial equilibrium shapes of a cable net with frictionless joints by parametrically varying the boundary shape. Bletzinger [21,22] presented a homotopy method for avoiding instability in finding the equilibrium shape corresponding to specified stresses. In his approach, however, only the equilibrium shape is considered, and it is not clear whether the curved surface can be generated by stretching a set of plane membrane sheets to the specified stress.

In this paper, a new method is presented for finding a series of initial equilibrium shapes of membrane structures supported by a frame, where the variable defining the boundary shape is considered as the parameter. The structure is modeled by the finite element method, and explicit developability conditions [23] are introduced for the curved surface to be reduced to a set of plane membrane sheets after removing the pretensioning force. By using the developability conditions, a quadratic programming problem is first formulated to directly find prestresses as well as the cutting patterns for specified coordinates of all the nodes of the finite element model. Then a trade-off design problem is defined for minimizing the weighted sum of the deviations of the initial stress and the shape from the desired distributions. The process of tracing the trade-off designs starts from the obvious solution where all the boundary nodes exist in a plane. An approximate method is presented for iterative correction of the solution based on the modified Newton–Raphson method without calculating the second order sensitivity coefficients of the stresses with respect to the nodal coordinates.

2. Finite element formulation of membrane

A membrane is discretized by using the triangular finite element model with constant strain [24]. Note that the method proposed in this paper may easily be applied to the case with other types of elements. Three nodes i, j, k of an element, and the local coordinate system $\mathbf{x} = (x, y)$ are defined as shown in Fig. 1 for the

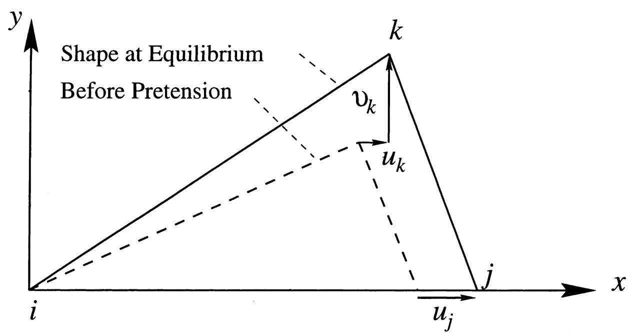

Fig. 1. Definition of the local coordinate system and the corresponding displacements.