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Preventive maintenance models with random maintenance quality

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Abstract

In real-world environments it is usually difficult to specify the quality of a preventive maintenance (PM) action precisely. This uncertainty makes it problematic to optimise maintenance policy. This problem is tackled in this paper by assuming that the quality of a PM action is a random variable following a probability distribution. Two frequently studied PM models, a failure rate PM model and an age reduction PM model, are investigated. The optimal PM policies are presented and optimised. Numerical examples are also given.

Keywords: Preventive maintenance; Long-run average cost; Failure rate; Maintenance quality

1. Introduction

Maintenance actions can generally be divided into two types: corrective maintenance (CM) and preventive maintenance (PM). The quality of maintenance actions in both CM and PM is an interesting research topic in the reliability literature, and is also vitally important when maintenance policies are being developed in practice.

The state of a piece of equipment after a maintenance action is performed is assumed to be one of the three situations: *perfect*, *imperfect*, and *minimal*. A *perfect* maintenance action is assumed to restore the equipment to be as good as new; an *imperfect* maintenance action may bring the equipment to any condition between as good as new and as bad as previously, and a *minimal* maintenance action is assumed to restore the equipment to a state the same as before the action. Examples of models for perfect, imperfect and minimal maintenance actions are *Renewal Processes*, *Generalized Renewal Processes* and *Non-Homogeneous Poisson Processes*, respectively. More comprehensive discussion in maintenance from both theoretical and application points of view can be found in Pham and Wang [1]; Wang [2] and Scarf [3].

The assumption that the equipment can be restored imperfectly, or imperfect maintenance, is closer to many practical scenarios than the other two assumptions. For modelling the quality of a PM action, two approaches have often been studied: a failure rate PM model by Lie and Chun [4] and Nakagawa [5,6], and an age reduction PM model by Canfield [7] and Malik [8]. Based on these two models, Lin et al. [9,10] introduced a hybrid PM model that combines the failure rate PM model and the age reduction PM model.

Assume that PM actions on the equipment are carried out at every time interval T independent of the failure history of the equipment, and CM actions are conducted upon failures. The failure rate PM model, the age reduction PM model and the hybrid PM model are defined as follows.

- *Failure Rate PM Model* [4–6]. The failure rate after the kth PM becomes $h_k(t)=\theta h_{k-1}(t)$ for $t\in(0,T)$, where $\theta(>1)$ is the adjustment factor, $h_k(t)$ $(t\in(0,T))$ is the failure rate after the kth PM, and T is the time interval between two adjacent PM actions. Each PM resets the failure rate to zero and the rate of increase of the failure rate gets higher after each additional PM. This model considers the change of the slope of the failure rate function. In this model, the adjustment factor θ is an index for measuring the quality of PM.
- *Age Reduction PM Model* [7,8]. Candield [7] and Malik [8] introduced age reduction models. In the age reduction model introduced by Canfield [7], the effective age after the kth PM reduces to $t_k-\eta$ if the equipment's effective age was t_k just prior to this PM, where $\eta(<t_k)$ is

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the restoration interval in the effective age of the equipment due to the kth PM. The restoration interval η in this model is an index for measuring the quality of PM. In the age reduction model introduced by Malik [8], the effective age after the kth PM reduces to bt_k if the equipment's effective age was t_k just prior to this PM, where $b<1$.

- *Hybrid PM Model* [9]. The failure rate after the kth PM becomes $a_k h(bt_k+x)$, where t_k is the time when the kth PM is conducted, $1=a_0\le a_1\le a_2\le,\ldots,a_{N-1}$, $0=b_0\ge b_1\ge b_2\ge,\ldots,b_{N-1}<1$, $x>0$ and $h(t)$ is the failure rate of the equipment when there is no CM or PM. Here, parameter a_k plays the same role as the parameter θ in the failure rate PM model, and parameter b_k functions similarly as the parameter b in the Malik's age reduction PM model.

All of the above three models assume that the failure rate of the equipment is increasing with time when no PM is conducted. This paper only studies the failure rate PM model and the Canfield age reduction PM model.

The parameters that determine the PM quality are the adjustment factor θ in the failure rate PM model and the restoration interval η in the Canfield age reduction PM model. They are important because they impact on the frequency of PM's, and therefore the long-run average cost. These parameters can be estimated based on a domain expert's suggestion [8] or real data [7]. It is assumed by prior research on the above two models that the parameters θ and η are fixed constant. This assumption may be violated in many scenarios, especially in the case when the parameters are estimated by domain experts. It can be more practical to assume that these two parameters are random variables following certain probability distributions. Most maintenance engineers in building service systems, for example, usually do not indicate that the restoration interval of a PM is 2 years, they tend to estimate the restoration interval falls within an interval (1, 3) years instead. In this case, it can assume that the restoration interval is a random variable with a uniform probability distribution.

This paper considers the scenarios when the maintenance quality is a random variable. It assumes that both the adjustment factor θ and the restoration interval η are random variables with certain probability distributions. Optimal PM policies for these two models are then obtained.

The paper is organized as follows. Section 2 introduces two novel PM models that consider failure rate PM models and age reduction PM models whose parameters for measuring the maintenance quality are random variables, and provides with algorithms for optimising PM policies. Further discussions on the quality of PM's are made in Section 3. Section 4 investigates two cases where the quality of PM's are assumed to be uniformly distributed and the failure time to be Weibull distributions. Finally, in the last section, concluding remarks are given.

2. Model formulations

In this section, we investigate the two PM models, the failure rate PM model and the Canfield age reduction PM model, when the adjustment factor and the restoration interval are random variables. Model A assumes that the failure rate of a piece of equipment after a PM action become a product of a maintenance quality and the failure rate before the PM action. Model B assumes that the equipment's age become younger than before a PM action.

2.1. Model A

2.1.1. Assumptions

(A) PM actions are performed at time kT for $k=1,\ldots,N$. The equipment is replaced at time NT, where T is the time interval between two adjacent PM actions and N is the number of PM actions before a replacement.

(B) When there is no PM or CM, the failure rate of the equipment, denoted by $h(t)$, is strictly increasing.

(C) The equipment has the failure rate $h_k(t)=\theta^{k-1}h(t)$ after the kth PM, where $t\in(0,T)$, and θ is a random variable with a cumulative distribution function, denoted by $F(\theta)$ with $\theta\ge 1$. Assume that the Nth moment about the random variable θ exists.

(D) A replacement can restore the equipment to be as good as new. The equipment undergoes minimal repair upon failures between two adjacent PM's. The failure rate remains unchanged by minimal repair.

(E) The times for PM, minimal repair and replacement are negligible.

(F) The costs on a minimal repair, a PM and a replacement are $c_{\rm m}$, $c_{\rm p}$ and $c_{\rm r}$, respectively.

The long-run average cost rate is

$$C_{\rm A}(T,N)=\frac{1}{NT}\left\{c_{\rm m}\sum_{k=1}^{N}\int_0^T\left(\int_1^\infty\theta\,{\rm d}F(\theta)\right)^{k-1}h(t){\rm d}t+(N-1)c_{\rm p}+c_{\rm r}\right\}\tag{1}$$

denote $\gamma_k=\left(\int_1^\infty\theta\,{\rm d}F(\theta)\right)^{k-1}$ and $r_k(t)=\gamma_k h(t)$. Eq. (1) can be re-written as

$$C_{\rm A}(T,N)=\frac{1}{NT}\left\{\left(c_{\rm m}\sum_{k=1}^{N}\int_0^T r_{\rm k}(t){\rm d}t\right)+(N-1)c_{\rm p}+c_{\rm r}\right\}\tag{2}$$

To determine the optimal values of N and T that minimize $C_{\rm A}(T,N)$ in Eq. (2), one can solve the following optimisation problem.

$$C_{\rm A}(T_0,N_0)=\min_{T,N}C_{\rm A}(T,N)\tag{3}$$