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Reliability Engineering and System Safety 91 (2006) 460–468

RELIABILITY ENGINEERING & SYSTEM SAFETY
www.elsevier.com/locate/ress

Periodic imperfect preventive maintenance with two categories of competing failure modes

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Received 21 April 2004; received in revised form 11 February 2005; accepted 17 March 2005
Available online 12 May 2005

Abstract

A maintenance policy is studied for a system with two types of failure modes: maintainable and non-maintainable. The quality of maintenance actions is modelled by its effect on the system failure rate. Preventive maintenance actions restore the system to a condition between as good as new and as bad as immediately before the maintenance action. The model presented permits to study the equipment condition improvement (improvement factor) as a function of the time of the preventive maintenance action. The determination of the maintenance policy, which minimizes the cost rate for an infinite time span, is examined. Conditions are given under which a unique optimal policy exists.

Keywords: Preventive maintenance; Minimal repair; Improvement factor; Maintainable failure modes; Non-maintainable failure modes

1. Introduction

The optimal determination of maintenance policies is an important issue in reliability engineering. Preventive maintenance actions may increase equipment lifetime and decrease in-service breakdowns frequency. Usually, it is assumed that preventive maintenance actions restore the system to a good as new condition. Nevertheless, this assumption does not hold always in practice. For example when only some of the components of a complex system are replaced the condition of the whole system can be considered to be between as good as new and as bad as immediately before the maintenance action [20].

One way of modelling imperfect maintenance actions is to consider that the component condition after a maintenance action is the same as immediately before the maintenance action (minimal repair) with probability p and as good as new (replacement) with probability $1-p$ [2]. Alternatively, the effect of the maintenance action can be modelled by using the system effective age or the failure rate function [10,18]. Malik [10] introduced improvement factors to model the effect of maintenance actions. Lie and Chun [8] and Nakagawa [14] considered improvement factors in failure rate function and effective age. Nakagawa [16] introduced improvement factors in hazard rate and age for a sequential preventive maintenance policy and analyzed two corresponding imperfect preventive maintenance models.

In this paper, the concept of failure mode is used. A definition of failure mode is given by Mosleh et al. [11]. They define a failure mode (page 110) as 'A description of component failure in terms of the component function that was actually or potentially unavailable'. Høyland and Rausand [6] (page 10) state that 'All technical items are designed to fulfill one or more functions. A failure mode is thus defined as non-fulfillment of one of these functions'. Accordingly, it will be written 'failure with respect to a given failure mode of the system' if the corresponding function is unavailable.

Usually maintenance actions such as oiling and cleaning or partial system replacement only restore equipment to a good as new condition with respect to some failure modes while increased failure-proneness of other failure modes due to wear, for example, is not eliminated. Lin et al. [9] modelled this phenomenon by introducing the concept of two categories of failure modes, maintainable failure modes

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doi:10.1016/j.ress.2005.03.009

Nomenclature

c_m	cost of a minimal repair
c_p	cost of a preventive maintenance action
c_r	cost of system replacement
$\Gamma(s)$	Gamma function, i.e. $\Gamma(s)=\int_0^\infty y^{s-1}e^{-y}\mathrm{d}y$,
t	time
T	decision variable: fixed period between preventive maintenance actions, $T>0$
T^*	optimal value of T
N	decision variable: after $N-1$, $N=2,3,\ldots$, imperfect preventive maintenance actions the next preventive maintenance action at time NT, $N=1,2,\ldots$, restores the system to a good as new condition
N^*	optimal value of N
$C(T,N)$	cost rate as a function of T and N
$C_\mathrm{C}(T,N)$	expected cost in a cycle as a function of T and N
δ	parameter used to specify δ_0
δ_0, p_0	parameters used to specify $p(t)$ in the form $p(t)=p_0+\delta_0\lambda(t)$, $0\le t<T$
$\gamma_k(T)$	improvement factor of the kth preventive maintenance action in a cycle for a given value of T, $k=1,2,\ldots,N-1$, $N=2,3,\ldots$
$h(t)$	hazard rate of non-maintainable failure modes
$H(t)$	cumulative hazard rate of non-maintainable failure modes
$\lambda(t)$	hazard rate of maintainable failure modes if $p_k(t)=0$, $t\ge 0$
$\Lambda(t)$	cumulative hazard rate of maintainable failure modes if $p_k(t)=0$, $t\ge 0$, $\Pi(k,T)=\int_{(k-1)T}^{kT} r_{k,T}(t)\mathrm{d}t$, $k=1,2,\ldots,N$, $N=1,2,\ldots$
$p_k(t)$	function which models the dependence between maintainable and non-maintainable models for t in the interval $((k-1)T,\ kT]$, $k=1,2,\ldots,N$, $N=1,2,\ldots$. When $p_k(t)$ does not depend on k it is written simply $p(t)$.
$r_{k,T}(t)$	maintained system failure rate for t in the interval $((k-1)T,\ kT]$, $k=1,2,\ldots,N$, $N=1,2,\ldots$

The maintained system failure rate is denoted as $r_{k,T}(t)$, $(k-1)T\le t<kT$, to stress its dependence on the decision variable T. To simplify the exposition the maintained system failure rate will be referred to as the system failure rate.

A cycle is defined as the period of time between two subsequent system replacements.

For a function $g(t)$, $g'(t)$ and $g''(t)$ will denote its first and second derivatives, respectively.

Throughout the paper, the terms increasing and decreasing will mean strictly increasing and strictly decreasing, respectively.

If it is not otherwise stated indexes k and N are understood to take values $k=1,2,\ldots,N$ and $N=1,2,\ldots$, respectively.

and non-maintainable failure modes, into the modelling of imperfect preventive maintenance activities. Similarly to Lin et al. [9], the system failure modes are divided into maintainable and non-maintainable failure modes. That is, it will be assumed that there are system functions (maintainable failure mode) for which the system degradation leading to its unavailability can be removed by preventive maintenance actions. Removing degradation related to other system functions (non-maintainable failure mode) is only possible by making a complete overhaul which restore the whole system to a good as new condition. Further it will be supposed that a failure rate function can be related with each failure mode. The approach presented in this paper differs from the one used by Lin et al. [9]: while they use adjustment factors in effective age and hazard rate, it is modelled explicitly the effect of preventive maintenance actions by the reduction of the failure rate of the maintainable failure mode.

The model presented in this paper is related to competing risks models. Maintainable and non-maintainable failure modes compete to provoke the system failure. Further three types of maintenance actions are considered: preventive (imperfect) maintenance, minimal repairs and replacements. For references on competing risks models see [4,19] and [22]. The dependence scheme obtained for competing risks for failures through the model of this paper can be considered as 'positive' in the sense that shorter times to failure with respect to non-maintainable failure models tend to occur together with shorter times to failure with respect to maintainable failure modes [1]. Modeling stochastic dependence is an ample subject. One useful tool in this regard is the copula [17].

The model presented in this paper can be applied to multi-component series systems in which some components are replaced frequently while others are replaced with a smaller frequency. Maintainable failure modes would correspond to frequently replaced components. Non-maintainable failure modes would correspond to less frequently replaced components. Even if a component is replaced frequently its degradation may depend on the degree of degradation of less frequently replaced components because of, for example, physical interactions like vibration or high temperature. Since the behavior of the failure rate can be used to characterize the system degradation, the dependence between maintainable and non-maintainable failure modes can be stated in terms of failure rates. In this paper, this approach is used, i.e. it is considered that the failure rate of maintainable failure modes depends on the failure rate of non-maintainable failure modes. Possible practical application can be made on systems like electric truck motors