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Optimal threshold value of failure-rate for leased products with preventive maintenance actions

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Abstract

This paper investigates the optimal threshold value of failure rates for leased products with a Weibull lifetime distribution. Within a lease period, any product failure is rectified by minimal repairs and the lessor may incur a penalty when the time required to perform a minimal repair exceeds a reasonable time limit. To reduce product failures, additional preventive maintenance actions are carried out when the failure rate reaches a threshold value. Under this maintenance scheme, a mathematical model of the expected total cost is established. Based on the model, the optimal threshold value and the corresponding maintenance degrees are derived such that the expected total cost is minimized. The structural properties of the optimal policy are investigated and an efficient algorithm is provided to search for the optimal policy. Finally, numerical examples are given to illustrate the features of the optimal policy.

Keywords: Preventive maintenance; Minimal repair; Threshold value; Leased product

1. Introduction

Due to the increase in complexity of products and rapid advances in technological innovation, there is a trend to lease a product rather than own a product. For a leased product, performing maintenance actions usually requires expensive equipment and professional technicians, which is not economical for the lessee (the one leasing the product). Therefore, the maintenance of the product is usually specified in a lease contract to ensure that the product could fulfill its intended purpose [1]. In this paper, we propose a maintenance scheme, in which preventive maintenance actions are taken when the failure rate of the leased product reaches a certain threshold value since this maintenance scheme can be easily specified in a lease contract in practice.

Maintenance actions usually can be classified into two major categories: (i) Corrective Maintenance (CM) and (ii) Preventive Maintenance (PM). CM actions are used to rectify a failed product back to its operational state, and PM actions are performed to improve the operational state of the product to avoid failures. For repairable products, various maintenance policies have been extensively discussed in the literature [2–5]. In practice, minimal repair is the most commonly used CM action to restore a failed product [6,7]. After minimal repair, the product is operational but

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the failure rate of the product remains unchanged. Various issues associated with minimal repair can be found in the literature [6–9].

To reduce the number of failures and possible penalties within the lease period, PM actions are usually taken by the lessor. Many PM policies have been proposed and studied under various situations, such as finite or infinite horizon [2,3], and perfect or imperfect maintenance [4,10,11]. Nakagawa [6] proposed an imperfect PM for repairable products in which the degree of PM is described by the reduction in age of the product. Since then, the age-reduction method has been widely adopted in the research of imperfect maintenance policies.

Another method for describing the degree of a preventive maintenance is the failure-rate reduction method (FRRM). In 1993, Chan and Shaw [12] proposed two methods of failure-rate reduction: (i) failure rate with fixed reduction; that is, the failure rate is reduced by a fixed quantity after each PM action; and (ii) failure rate with proportional reduction; that is, the failure rate is reduced by an amount proportional to the current failure rate. In this paper, we will adopt the fixed failure-rate-reduction method to describe the degree of PM and derive the optimal PM policy for a leased product, since this method can be clearly specified in a lease contract.

The remainder of this paper is organized as follows. The mathematical model is developed in Section 2 for the case when the failure density is Weibull. The properties of the optimal PM policy are investigated in Section 3. In Section 4, the impact of providing preventive maintenance within a lease period is illustrated through numerical examples. Finally, some conclusions are drawn in the last section.

2. Mathematical formulation

Consider that a new product with Weibull lifetime distribution is leased for a period of L. It is well-known that the probability density function of Weibull is given by $f(t) = \lambda\beta(\lambda t)^{\beta-1}e^{-\lambda t}$ for $t \geq 0$, where $\lambda > 0$ is the scale parameter and $\beta > 0$ is the shape parameter. Since Weibull distributions can provide a versatile class of distributional forms by changing parameter values, it is one of the most commonly used distributions in reliability engineering. By the definition of a failure rate function, the failure rate function of the Weibull distribution is $r(t) = \lambda\beta(\lambda t)^{\beta-1}$ and its inverse function is given by $r^{-1}(t) = [t/(\lambda^{\beta}\beta)]^{1/(\beta-1)}$. Note that both $r(t)$ and $r^{-1}(t)$ increase in t if $\beta > 1$, and decrease in t if $\beta < 1$. When $\beta = 1$, $r(t) = \lambda$ is a constant but $r^{-1}(t)$ does not exist. In this paper, we focus on the case where the failure rate $r(t)$ continuously increases in t and its inverse function exists (i.e. $\beta > 1$) within the lease period.

Within the lease period, any failure of the leased product is rectified by minimal repairs. Each minimal repair incurs a fixed repair cost $C_r > 0$ to the lessor and requires a random amount of repair time t_r that follows a general cumulative distribution function $G(t_r)$. If the repair time exceeds a pre-specified time limit τ, then there is a penalty C_τ to the lessor. After minimal repair, the product is operational, but the failure rate of the product remains the same as that just before failure.

To reduce the number of failures within the lease period, imperfect PM actions with degree $\delta \geq 0$ are carried out whenever the failure rate of the product reaches a threshold value θ. That is, the failure rate of the product is reduced by $\delta \leq \theta$ after each PM, where δ reflects the effect of various preventive maintenance actions on the leased product. Based on prior experience or analytical results from maintenance data, each maintenance action may result in a certain amount of failure rate reduction. Hence, once δ is obtained, the corresponding maintenance action can be determined. In general, the cost to perform an imperfect PM is a non-negative and non-decreasing function of the maintenance degree δ. Let $C_p(\delta)$ be the cost for performing a PM action with degree $\delta \geq 0$. Then, we have $C_p(\delta) \geq 0$ and $C'_p(\delta) \geq 0$ for all $\delta \geq 0$. In this paper, we consider the case where the PM cost is a linearly increasing function of the maintenance degree; that is, $C_p(\delta) = a + b\delta$ for any $a > 0$ and $b \geq 0$. Assuming that the time required for performing an imperfect PM is negligible, the preventive maintenance scheme can be described in terms of the failure rate function as shown in Fig. 1.

Under the aforementioned maintenance scheme, the acronyms and mathematical notations used in this paper are summarized as follows.

CM	Corrective Maintenance
PM	Preventive Maintenance
FRRM	Failure-Rate Reduction Method
$f(t)$	probability density function of the Weibull distribution, that is, $f(t) = \lambda\beta(\lambda t)^{\beta-1}e^{-\lambda t}$ for $t \geq 0$, where $\lambda > 0$ is the scale parameter and $\beta > 0$ is the shape parameter