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Modelling and optimizing sequential imperfect preventive maintenance

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ABSTRACT

This paper deals with the problem of scheduling imperfect preventive maintenance (PM) of some equipment. It uses a model due to Kijima in which each application of PM reduces the equipment's effective age (but without making it as good as new). The approach presented here involves minimizing a performance function which allows for the costs of minimal repair and eventual system replacement as well as for the costs of PM during the equipment's operating lifetime. The paper describes a numerical investigation into the sensitivity of optimum schedules to different aspects of an age-reduction model (including the situation when parts of a system are non-maintainable—i.e., unaffected by PM).

1. Introduction

Most organizations incur significant costs associated with equipment failure and its subsequent repair or replacement. The frequency of such failure can typically be reduced by periodic maintenance. Mathematical models for analysing and optimizing the performance of repairable equipment have been widely discussed in the literature [1–14]. In this paper we follow ideas given in [1,8] and study the optimal scheduling of preventive maintenance (PM), basing our approach on the notion that equipment which benefits from PM can have an *effective age* which is less than its calendar age.

When only minimal repairs are performed and there are no other interventions, the likelihood of equipment failure can be expected to increase steadily with time. More precisely, we suppose that the number of failures occurring during a time interval (a, b) is

$$\int_a^b h(t)\,\mathrm{d}t.$$

The function $h(t)$ is sometimes called the *failure rate* or *hazard rate* (as in [8,10]) and sometimes the *failure intensity* [4,5]. If $H(t)$ denotes the indefinite integral $\int_0^t h(s)\,\mathrm{d}s$, the number of failures occurring between $t = a$ and $t = b$ is $H(b) - H(a)$. $H(t)$ is called the *cumulative failure rate*.

In practice, PM is used to lengthen the useful lifetime of equipment (and hence to decrease average running cost) by reducing the occurrence of failures. One of the key characteristics of a maintenance model is the effect of different kinds of intervention on the age of the system. *Perfect repair* and *minimal repair* are both commonly used in idealized age-effect models; and similar terms can also be applied to maintenance. In reality, however, both repair and maintenance are usually *imperfect*—i.e., somewhere between perfect and minimal. Pham and Wang [11] and, more recently, Doyen and Gaudoin [4] have given useful surveys of imperfect maintenance models. One of the most important of these is the *effective age model* [6,7]. This is also called the virtual age model. If we assume that maintenance makes the equipment's effective age, y, less than its calendar age, t, then the number of failures occurring after a PM will depend on $H(y)$ rather than $H(t)$. Since H is a monotonically increasing function, fewer failures will occur after a PM than if PM had not been carried out.

Our purpose in this paper is to consider the optimal scheduling of PM. Our particular focus is on the way that such schedules can be affected by the choice of aging model that is used. Specifically, we compare the so-called types 1 and 2 aging models proposed in [6,7]. These have also been recently discussed in an optimization context by Kahle [5].

2. Effective age models

In what follows we shall use x_k to denote the interval between the $(k-1)$-th and k-th PM. Thus, if equipment enters service at

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time $t = 0$, the first PM occurs at time $t_1 = x_1$. Just *before* this maintenance, the effective age y_1 is the same as its calendar age x_1. Immediately after PM, however, the effective age is reduced to b_1x_1, where b_1 is some constant $(0<b_1<1)$. (We note here that we are making the idealized assumption that time taken to perform PM is negligible. We would argue that this is not an unduly restrictive feature of the model but we shall discuss it more fully in the final section of the paper.) Then, until the next PM at time $t_2 = x_1 + x_2$, the effective age is $y = b_1x_1 + x$ for $0<x<x_2 = t_2 - t_1$. In particular, we denote the effective age just *after* the first PM by $y_1^+ = b_1x_1$. The effective age of the system just before the second PM at time t_2 is then $y_2 = b_1x_1 + x_2$.

After the second (and subsequent) PMs, the effective age reduction can be modelled in two different ways [2,7]. In *type 1 effective age reduction* [7] it is assumed that, immediately after the second PM, the effective age becomes

$$y_2^+ = y_1^+ + b_2x_2 = b_1x_1 + b_2x_2 = y_2 - (1 - b_2)x_2$$
where $0<b_1\leqslant b_2<1$.

More generally, between the $(k-1)$-th and the k-th PM, the effective age is

$$y = b_{k-1}x_{k-1} + \cdots + b_1x_1 + x, \tag{1}$$

where $0<x<x_k = t_k - t_{k-1}$ and $b_1 \leqslant b_2 \leqslant \cdots \leqslant b_k \leqslant 1$. Thus the effective age immediately after the $(k-1)$-th PM is

$$y_{k-1}^+ = y_{k-1} - (1 - b_{k-1})x_{k-1}. \tag{2}$$

In *type 2 effective age reduction* [7] it is assumed that the effective age immediately after the second PM is $y_2^+ = b_2y_2 = b_2b_1x_1 + b_2x_2$. More generally, between the $(k-1)$-th and the k-th PM, the effective age is

$$y = b_{k-1}y_{k-1} + x = (b_{k-1}\cdots b_2b_1)x_1 + \cdots + b_{k-1}x_{k-1} + x, \tag{3}$$

where $0<x<x_k = t_k - t_{k-1}$ and $b_1 \leqslant b_2 \leqslant \cdots \leqslant b_k \leqslant 1$. In particular, the effective age just after the $(k-1)$-th PM is

$$y_{k-1}^+ = b_{k-1}y_{k-1}. \tag{4}$$

More compactly, (3) can be written $y = B_{k-1}x_{k-1} + B_{k-2}x_{k-2} + \cdots + B_1x_1 + x$, where $0<x<x_k = t_k - t_{k-1}$ and B_j denotes the product $b_{k-1}b_{k-2}\cdots b_j$.

Type 1 age reduction has been investigated by Kijima and co-workers in [6,7] while the type 2 model has also been discussed by Dagpunar [2] and Lin et al. [8]. Both types are considered in the recent papers by Doyen and Gaudoin [4] and Kahle [5]. The main difference between these effective age-reduction models is as follows. In the type 1 model, the k-th PM makes an effective age reduction *only* as regards the actual aging of the system since the $(k-1)$-th PM. In the type 2 model, however, each PM is assumed to cause an effective decrease in *all* the aging that has taken place since time $t = 0$. Hence, under the type 2 model, repeated PMs can have a cumulative age-reduction effect which does not occur with the type 1 model.

We could say that the type 2 model takes a more optimistic view of the benefits of PM. If we suppose that PM occurs annually then, for both the types 1 and 2 aging models, the effective age after maintenance at the end of the first year is $b_1(<1)$ years. Hence both models predict the same number of failures in year two. However, the effect of PM at the end of year two is dependent on which aging model is used. The type 1 effective age after the second PM is $b_1 + b_2$ years; but the type 2 effective age is $(b_1b_2 + b_2)$ years. If $b_1 = b_2 = 0.5$, say, then the types 1 and 2 effective ages after the second PM are, respectively, 1 year and 0.75 years. Thus, during year three, the type 2 model implies fewer than the type 1 model. This difference will become even more marked in subsequent years.

Extra parameters can be included in both age-reduction models to make them reflect the complexities of a real system. In practice, after a number of PMs have been performed, equipment may be less robust than its effective age suggests; and we can model this using variable scaling factors on $h(t)$. We suppose the number of failures occurring in $(0, t_k)$ can be written as

$$H(t_k) = \int_0^{y_1} h(y)\,\mathrm{d}y + \sum_{j=1}^{k-1}\left\{\int_{y_j^+}^{y_{j+1}} A_j h(y)\,\mathrm{d}y\right\}$$
$$= \int_0^{y_1} h(y)\,\mathrm{d}y + \sum_{j=2}^{k} H_{j-1}, \tag{5}$$

where

$$H_{j-1} = \int_{y_{j-1}^+}^{y_j} A_{j-1}h(y)\,\mathrm{d}y \tag{6}$$

and where the A_j are constants such that $1 \leqslant A_1 \leqslant A_2 \leqslant \cdots$. (Expression (5) can also be extended to make a distinction between *maintainable* and *non-maintainable* failure modes (see [8]). This will be considered in a later section.)

The PM schedules presented in [8] are optimized by a semi-analytic solution technique which takes advantage of the relatively simple forms (Weibull functions) chosen for the function $h(t)$. Bartholomew-Biggs et al. [1] have considered the same type 2 age-reduction model as used in [8] but their PM schedules are optimized using general-purpose nonlinear minimization algorithms. Such techniques may be more suitable when the $h(t)$ are more complicated than Weibull functions. The problem formulations in [1] also feature constraints to exclude spurious solutions with unacceptably short (or even negative) intervals between PM. The main purpose of the present paper is to compare types 1 and 2 age-reduction models when used to determine optimum PM schedules in a framework similar to that described in [1].

Before we proceed to formulate an optimization problem we shall list some general assumptions and notation.

General assumptions:

The system enters service at time $t = 0$.
When a system failure occurs, minimal repair takes place instantly.
PM is completed instantly.
The system may have two categories of failure modes, i.e., maintainable and non-maintainable.
The failure rate for non-maintainable parts of the system is not affected by minimal repair, PM or system failure.
The failure rate for maintainable parts of the system is not changed by minimal repair but it is changed whenever a PM is performed.

Notation:

t_k	time duration from $t = 0$ to the time of the k-th PM
$x_k = t_k - t_{k-1}$	interval between the $(k-1)$-th and k-th PM
y_k	effective age of the system just before k-th PM
y_k^+	effective age of system just after k-th PM
N	total number of PM performed. (*The N-th PM is a system replacement*)
$h_a(t)$	failure rate of maintainable components
$h_b(t)$	failure rate of non-maintainable components
$H(t)$	cumulative failures up to time t *when no PM occurs*. See (16)
A_k	adjustment factor on $h_a(t)$ due to the k-th PM. $A_k \geqslant 1$
b_k	effective age-reduction factor due to the k-th PM. $b_k \leqslant 1$