Journal of Environmental Economics and Management 59 (2010) 142–148

Contents lists available at ScienceDirect

Journal of Environmental Economics and Management

journal homepage: www.elsevier.com/locate/jeem

Expected net present value, expected net future value, and the Ramsey rule

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ARTICLE INFO

Article history:
Received 17 November 2008
Available online 1 December 2009

Keywords:
Discount rate
Ramsey rule
Climate change
Cost–benefit analysis

ABSTRACT

Weitzman [1] showed that when future interest rates are uncertain, using the expected net present value implies a term structure of discount rates that is decreasing to the smallest possible interest rate. On the contrary, using the expected net future value criteria implies an increasing term structure of discount rates up to the largest possible interest rate. We reconcile the two approaches by introducing risk aversion and utility maximization. We show that if the aggregate consumption path is optimized and made flexible to news about future interest rates, the two criteria are equivalent. Moreover, they are also equivalent to the Ramsey rule extended to uncertainty.

1. Introduction

The long term sustainability of economic growth is a crucial topic for public opinions. Still, there is much disagreement in academia about how future generations should be treated in the evaluation of public policies. Indeed, the classical cost–benefit methodology is often criticized for sacrificing distant generations. This raises questions about the level and term structure of the discount rate. There is no consensus in our profession about the rate that should be used to discount long term costs and benefits. This implies that economists disagree fundamentally about the intensity of our effort to improve the distant future. This is best illustrated in the field of climate change, but other applications exist in relation to nuclear wastes, biodiversity, genetically modified organisms, or to the preservation of natural resources. Stern [2], who implicitly uses a discount rate of 1.4% per year, estimated the current social cost of carbon around 85 dollars per ton of CO_2. Nordhaus [3] criticizes the low discount rate used by Stern, and recommend alternatively a discount rate of 5%, which leads to an estimation of 8 dollars per ton of CO_2. This huge difference in the estimation of the social cost of carbon yields radical discrepancies in the policy recommendations related to global changes. This is due to the exponential nature of the impact of a change of the discount rate on net present values (NPV). For example, using a rate of 5% rather than 1.4% to discount a cash flow occurring in 200 years reduces its NPV by a factor 1340.

A standard arbitrage argument justifies using the rate of return of capital, hereafter denoted ρ, in the economy as the socially efficient discount rate. Indeed, diverting productive capital to invest in environmental projects with an internal rate of social return below ρ would reduce the welfare of future generations. Another argument is based on the NPV. At equilibrium, the interest rate in the economy equals ρ. Suppose that one borrows today at rate ρ an amount equal to the

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[1] This paper benefited from the financial support of the Chair “Sustainable finance and responsible investment” at TSE. The research leading to these results has also received funding from the European Research Council under the European Community’s Seventh Framework Programme (FP7/2007-2013) Grant Agreement no. 230589.

0095-0696/$ - see front matter
doi:10.1016/j.jeem.2009.11.003

present value of future cash flows, which implies that these cash flows will be just enough to reimburse the initial loan. This converts all future costs and benefits into present costs and benefits. Thus, the project should be implemented only if its NPV discounted at rate ρ is positive. However, this argument requires knowing the returns of capital for the different maturities of the cash flows of the environmental projects under scrutiny. Obviously, these future returns are highly uncertain for the time horizons that we have in mind when we think about climate change or nuclear waste.

Weitzman [1,4] has developed a simple argument based on this fact to recommend a smaller rate to discount distant impacts. The NPV is a decreasing convex function of the interest rate, and the degree of convexity of this function is increasing in the time horizon. Therefore, by Jensen's inequality, introducing an uncertain permanent shock on future interest rates raises the expected NPV, and this effect is increasing in the maturity. It implies that the uncertainty on ρ has an effect on the expected NPV that is equivalent to a sure reduction in ρ. It also implies that the term structure of this reduction is increasing. Newel and Pizer [5] and Groom et al. [6] estimated the impact of the uncertainty about future interest rates on the socially efficient discount rate for different time horizons. More recently, Gollier et al. [7] estimated the term structure based on the uncertainty of the GDP-weighted world interest rates. They obtained discount rates of 4.2%, 2.3% and 1.8%, respectively, for the short term, 100 years and 200 years.

We compare this approach with two alternative approaches. The first alternative approach is exactly symmetric to the one proposed by Weitzman [1]. It measures the impact of the uncertainty on the expected net *future* value (NFV), as described for example by Gollier [8], Hepburn and Groom [9], Freeman [10] and Buchholz and Schumacher [11]. Rather than assuming that all costs and benefits of the environmental project are transferred to the present, it assumes that they are all transferred to the terminal date of the project. Under this alternative approach the uncertainty on future interest rates raises the discount rate in an increasing way with the time horizon. The second alternative approach consists in making no intertemporal transfer of costs and benefits, which implies that consumption is modified to compensate them in real time. Under this approach, one needs to compare the impact of uncertain changes in consumption at different dates on the intertemporal welfare. Ramsey [12] derived a simple formula that equalizes the efficient discount rate net of the rate of pure preference for the present to the product of the growth rate of GDP by the index of aversion to (intertemporal) consumption inequality. Gollier [13,14] and Weitzman [15] extended this formula to take account of the uncertainty on the growth rate of GDP, which is itself linked to the uncertainty of future interest rates. They show that the term structure of the efficient discount rate is flat when shocks on the growth rate are temporary, and is decreasing when shocks have a permanent component.

These results appear to conflict with one another. The aim of this paper is to unify them in a single framework under uncertainty. We show that these approaches are equivalent once risk aversion is properly integrated into the model. We make explicit the investor's objective function, which is assumed to be the standard expected utility criterion. We show how to generalize the discount rates based on the NPV and NFV rules to the case of risk aversion. In fact, we show that the expectations considered above must be based on risk-neutral probabilities, which are proportional to the marginal utility of consumption at the evaluation date. This is in line with the standard consumption-based methodology in finance to estimate risk premia.

Introducing risk aversion alone does not solve the puzzle. One must also require the consumption path is optimized. Under the condition that the investor optimizes her consumption plan contingent to the observed interest rate, we show that the NPV and NFV approaches lead to exactly the same term structure of discount rates, which is decreasing and tends to the lowest possible interest rate. Moreover, we show that the two approaches are also perfectly compatible with the well-known Ramsey rule.

This paper is mostly non-technical, in contrast to Gollier [16]. This companion paper also shows that the decreasing nature of the term structure obtained in this framework depends heavily upon the assumption that shocks are permanent. If they are purely transitory, the term structure of discount rates should be flat.

2. Net present value and net future value under uncertainty

Consider a simple investment that generates a sure payoff Z at date t per euro invested at date 0.[2] If ρ is the continuously compounded interest rate during the period, it is optimal to undertake the project if its net present value

$$NPV = -1 + Ze^{-\rho t}$$

is positive. This NPV rule is sustained by a simple arbitrage argument: implementing the project and borrowing $Ze^{-\rho t}$ at date 0 until date t would generate the sure payoff *NPV* today, with no other net payoff along the lifetime of the project. Suppose now that the interest rate $\tilde{\rho}$ that will prevail between dates 0 and t is constant but uncertain.[3] It is unknown at the time the investment decision must be made, but the uncertainty in $\tilde{\rho}$ is fully resolved at date $t = 0$. Because the investment opportunity cost is uncertain, it is likely to affect the optimal decision. Weitzman [1,4] assumes that the optimal decision criterion in that context is to invest if the expected NPV is positive, i.e., if $-1 + ZEe^{-\tilde{\rho} t}$ is positive. Obviously, this is

[2] Any sure investment project can be decomposed into a portfolio of investment projects with a single future cash flow occurring at different dates. So this assumption is made without loss of generality.

[3] We use tildes over variables when they represent random variables. When this symbol is removed, this represents a specific realization of the random variable.