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# Expected present value of total dividends in a delayed claims risk model under stochastic interest rates

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ABSTRACT

In this paper, a compound binomial risk model with a constant dividend barrier under stochastic interest rates is considered. Two types of individual claims, main claims and by-claims, are defined, where every by-claim is induced by the main claim and may be delayed for one time period with a certain probability. In the evaluation of the expected present value of dividends, the interest rates are assumed to follow a Markov chain with finite state space. A system of difference equations with certain boundary conditions for the expected present value of total dividend payments prior to ruin is derived and solved. Explicit results are obtained when the claim sizes are $K_n$ distributed or the claim size distributions have finite support. Numerical results are also provided to illustrate the impact of the delay of by-claims on the expected present value of dividends.



## 1. Introduction

In reality, insurance claims may be delayed due to various reasons. Since the work by Waters and Papatriandafylou (1985), risk models with this special feature have been discussed by many authors in the literature. For example, Yuen and Guo (2001) studied a compound binomial model with delayed claims and obtained recursive formulas for the finite time ruin probabilities. Xiao and Guo (2007) obtained the recursive formula of the joint distribution of the surplus immediately prior to ruin and deficit at ruin in this model. Xie and Zou (2008) studied a risk model with delayed claims. Exact analytical expressions for the Laplace transforms of the ruin functions were obtained. Yuen et al. (2005) studied a risk model with delayed claims, in which the time of delay for the occurrence of a by-claim is assumed to be exponentially distributed. A framework of delayed claims is built by introducing two kinds of individual claims, namely main claims and by-claims, and allowing possible delays of the occurrences of by-claims.

Dividend strategy for insurance risk models were first proposed by De Finetti (1957) to reflect more realistically the surplus cash flows in an insurance portfolio, and he found that the optimal strategy must be a barrier strategy. From then on, barrier strategies have been studied in a number of papers and books. For example, Claramunt et al. (2003) calculated the expected present value of dividends in a discrete time risk model with a barrier dividend strategy. Dickson and Waters (2004) showed how to use the compound binomial risk model to approximate the classical compound Poisson risk model in calculating the moments of discounted dividend payments. Other risk model involving dividend payments were studied by Zhou (2005), Gerber and Shiu (2004), Li and Garrido (2004), Wu and Li (2006), Frosting (2005), Bara et al. (2008) and the references therein.

All risk models described in the paragraph above relied on the assumption that the force of interest or the discount factor per period is a constant. Based on this assumption, it becomes evident that the discount factor per period embedded into the risk model fails to capture the uncertainty of the (future) risk-free rates of interest. In the compound binomial model with delayed claims and a dividend barrier proposed in this paper, discount factors are defined via the modelization of the one-period interest rates using a time-homogeneous Markov chain with a finite state space. The use of time-homogeneous Markov chains to model the interest rates is well documented in finance (see, e.g. Landriault (2008) in a discrete time framework). We derive the explicit expression for the expected present value of total dividends in our risk model.

The model proposed in this paper is a generalization of compound binomial risk model with paying dividends and classical risk model with delayed claims. It seems to be the first risk model with delayed claims and a constant dividend barrier in a financial market driven by a time-homogeneous Markov chain. We show that, the explicit expression for the expected present value of total dividends in this risk model can be obtained. The work of this paper can be seen as a complement to the work of Li (2008) that

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calculated the present value of total dividends in the compound binomial model under stochastic interest rates and extend the results of Li (2008) by introducing two kinds of individual claims, namely main claims and by-claims, and allowing possible delays of the occurrences of by-claims. The model considered in this paper is also related to the one considered by Wu and Li (2006). Although both models employ a discrete time risk model with dividends and delayed claims, our model differs from the one by Wu and Li (2006) as follows. Our model is more general than that of Wu and Li (2006) in that we assume the financial market is driven by a time-homogeneous Markov chain, while the discount factor per period is a constant in Wu and Li (2006).

It is obvious that the incorporation of the delayed claim and dividend payments makes the problem more interesting. It also complicates the evaluation of the expected present value of dividends. Because of the certainty of ruin for a risk model with a constant dividend barrier, the calculation of the expected discounted dividend payments is a major problem of interest (see, e.g. Wu and Li (2006)). Similar to method of Li (2008), we use the technique of generating functions to calculate the expected present value of total dividends for this risk model. Section 2 defines the model of interest, describes various payments, including the premiums, claims and dividends, and lists the notation. In Section 3, difference equations with certain boundary conditions are developed for the expected present value of total dividend payments prior to ruin. Then an explicit expression is derived, using the technique of generating functions. Moreover, closed-form solutions for the expected present value of dividends are obtained for two classes of claim size distributions in Section 4. Numerical examples are also provided to illustrate the impact of the delay of by-claims on the expected present value of dividends in Section 4.

## 2. The model

Here, we consider a discrete time model which involves two types of insurance claims; namely the main claims and the by-claims. Denote the discrete time units by $k = 0, 1, 2, \ldots$. In any time period, the probability of having a main claim is $q$, $0 < q < 1$. The occurrences of main claims in different time periods are independent. It is assumed that each main claim induces a by-claim. The by-claim and its associated main claim may occur simultaneously with probability $\theta$, or the occurrence of the by-claim may be delayed to the next time period with probability $1-\theta$. All claim amounts are independent, positive and integer valued. The main claim amounts $X_1, X_2, \ldots$, are independent and identically distributed. Put $X = X_1$. Then the common probability function of the $X_i$ is given by $f_m = \Pr(X = m)$ for $m = 1, 2, \ldots$. The corresponding probability generating function and mean are $\tilde{f}(s) = \sum_{m=1}^{\infty} f_m s^m$ and $\mu_X = \sum_{n=1}^{\infty} n f_n$, respectively. Let $Y_1, Y_2, \ldots$ be the independent and identically distributed by-claim amounts and put $Y = Y_1$. Denote their common probability function by $g_n = \Pr(Y = n)$ for $n = 1, 2, \ldots$, and write the probability generating function and mean as $\tilde{g}(s) = \sum_{n=1}^{\infty} g_n s^n$ and $\mu_Y$, respectively.

Let $S_k$ be the total amount of claims up to the end of the $k$th time period, $k \in \mathrm{N}^+$ and $S_0 = 0$. We define

$$S_k = S_k^X + S_k^Y, \tag{2.1}$$

where $S_k^X$ and $S_k^Y$ are the total main claims and by-claims, respectively, in the first $k$ time periods.

Similar to the model of Wu and Li (2006), we assume that premiums are received at the beginning of each time period with a constant premium rate of 1 per period, and all claim payments are made only at the end of each time period. We introduce a dividend policy to the company that certain amount of dividends will be paid to the policyholder instantly, as long as the surplus of the company at time $k$ is higher than a constant dividend barrier $b\,(b > 0)$. It implies that the dividend payments will only possibly occur at the beginning of each period, right after receiving the premium payment. The surplus at the end of the $n$th time period, $U_b(n)$, is then defined to be, for $n = 1, 2, \ldots$,

$$U_b(n) = u + n - S_n - D(n), \quad (U_b(0) = u), \tag{2.2}$$

where $D(n)$ is the sum of the total dividend payments in the first $n$ periods, with the definition

$$D(n) = D_1 + D_2 + \cdots + D_n, \quad (D(0) = 0),$$

with

$$D_n = \max\{U_b(n-1) + 1 - b, 0\}$$

being the amount of dividend paid out at the end of period $n$. Here the initial surplus is $u$, $u = 0, 1, \ldots, b$.

The positive safety loading condition holds if $q(\mu_X + \mu_Y) < 1$. Define $T_{u,b} = \inf\{n \in \mathrm{N} : U_b(n) < 0\}$ to be the time of ruin.

In this note, we assume that the interest rates $\{R_n, n \in \mathrm{N}\}$ with $R_n$ being the interest rate in the interval $(n, n+1]$ follow a time-homogeneous Markov chain with finite state space $\{r_1, r_2, \ldots, r_m\}$. The one-period transition probability matrix is given by $\mathbf{P} = (p_{i,j})_{i,j=1}^m$, where $p_{i,j} = \Pr(R_{n+1} = j | R_n = i)$, for $n \in \mathrm{N}$. The one-period discount factors are denoted by $v_1, v_2, \ldots$, respectively, where $v_i = 1/(1+r_i)$.

Under the interest rate model described above, the present value of total dividends until time of ruin $T_{u,b}$ given that the initial surplus is $u$ is denoted by

$$D_{u,b} = \sum_{k=1}^{T_{u,b}} D_k \left( \prod_{i=1}^{k} \frac{1}{1+R_i} \right), \quad u = 0, 1, 2, \ldots, b.$$

Define

$$V_i(u; b) = \mathrm{E}\left[D_{u,b} | R_0 = i\right], \quad i = 1, 2, \ldots, m,\ u = 0, 1, 2, \ldots, b$$

to be the expected present value of total dividend payments up to the time of ruin given that the initial interest rate $R_0 = r_i$.

The aim of this paper is to calculate $V_i(u; b)$, the expected present value of a sequence of dividend payments until the time of ruin under stochastic interest rates to the dividends, for some special claim size distributions so as to determine whether the optimal dividend barrier level is still independent of the initial surplus and the impact of the delay of by-claims on the expected present value of dividends.

## 3. A system of difference equations with boundary conditions

To study the expected present value of the dividend payments, $V_i(u; b)$, we need to study the claim occurrences in two scenarios (see Yuen and Guo (2001)). The first is that if a main claim occurs in a certain period, its associated by-claim also occurs in the same period. Hence there will be no by-claim in the next time period and the surplus process really gets renewed. The second is simply the complement of the first scenario. In other words, if there exists a main claim, its associated by-claim will occur one period later. Conditional on the second scenario, that is, the main claim occurred in the previous period and its associated by-claim will occur at the end of the current period, we define the corresponding surplus process as

$$U_b^*(n) = u + n - S_n - D^*(n) - Y, \quad n = 1, 2, \ldots, \tag{3.1}$$

with $U_b^*(0) = u$, where $D^*(n)$ is the sum of dividend payments in the first $n$ time periods, and $Y$ is a random variable following the probability function $g_n$, $n = 1, 2, \ldots$, and is independent of all other claim amounts random variables $X_i$ and $Y_j$ for all $i$ and $j$. The corresponding expected present value of the dividend payments