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Validating forecasts of the joint probability density of bond yields: Can affine models beat random walk?

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Abstract

Most existing empirical studies on affine term structure models (ATSMs) have mainly focused on in-sample goodness-of-fit of historical bond yields and ignored out-of-sample forecast of future bond yields. Using an omnibus nonparametric procedure for density forecast evaluation in a continuous-time framework, we provide probably the first comprehensive empirical analysis of the out-of-sample performance of ATSMs in forecasting the joint conditional probability density of bond yields. We find that although the random walk models tend to have better forecasts for the conditional mean dynamics of bond yields, some ATSMs provide better forecasts for the joint probability density of bond yields. However, all ATSMs considered are still overwhelmingly rejected by our tests and fail to provide satisfactory density forecasts. There exists room for further improving density forecasts for bond yields by extending ATSMs.

JEL classification: C4; C5; G1

Keywords: Density forecast; Affine term structure models; Probability integral transform; Financial risk management; Value at risk; Fixed-income portfolio management

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1. Introduction

The term structure of interest rates, which concerns the relationship among the yields of default-free bonds with different maturities, is one of the most widely studied topics in economics and finance. Following the pioneering works of Vasicek (1977) and Cox et al. (1985), a large number of multifactor dynamic term structure models (DTSMs) have been developed over the last two decades.[1] These models, by imposing cross-sectional and time series restrictions on bond yields in an internally consistent manner, provide important insights for our understanding of term structure dynamics. They have been widely used in financial industry for pricing fixed-income securities and managing interest rate risk.

Affine term structure models (ATSMs), first introduced in Duffie and Pan (1996), have become the leading DTSMs in the literature due to their rich model specification and tractability. In ATSMs, the short-term interest rate is an affine function of the underlying state variables which follow affine diffusions (the instantaneous drift and variance are affine functions of the state variables) under both the risk-neutral and physical measures. These assumptions allow closed-form solutions for a wide variety of fixed-income securities (see, e.g., Duffie et al., 2000; Chacko and Das, 2002), which greatly simplify empirical implementations of ATSMs. As a result, ATSMs have become probably the most widely studied DTSMs in the academic literature.

Despite the numerous empirical studies on DTSMs, the existing literature has mainly focused on in-sample fit of historical bond yields and ignored out-of-sample forecast of future bond yields. In-sample diagnostic analysis is important and can reveal useful information on possible sources of model misspecifications. However, it is the evolution of the yield curve in the future, not the past, that is most relevant in many financial applications, such as pricing and hedging fixed-income securities and managing interest rate risk. As widely recognized in the literature, the current yield curve contains information about the future yield curve and the state of the economy. Therefore, accurate forecasts of bond yields are also important for savings and investments decisions of households and firms, and for macroeconomic policy decisions of monetary authorities. Furthermore, as pointed out by Duffee (2002, p. 465), "a model that is consistent with finance theory and produces accurate forecasts can make a deeper contribution to finance," especially for explaining time varying expected bond returns and the failure of the expectation hypothesis.

Unfortunately, there is no guarantee that a model that fits historical data well will also perform well in out-of-sample forecast, due to at least three important reasons. First, the extensive search for more complicated models using the same (or similar) data set(s) may suffer from the so-called data snooping bias, as pointed out by Lo and MacKinlay (1989) and White (2000). In the present context, most studies on

[1] The theoretical and empirical literature on multi-factor dynamic term structure models is too huge and diversed to be summarized here. For excellent reviews of the current literature, see Dai and Singleton (2003) and Piazzesi (2004).