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Can interest rate volatility be extracted from the cross section of bond yields? ☆

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ABSTRACT

Most affine models of the term structure with stochastic volatility predict that the variance of the short rate should play a 'dual role' in that it should also equal a linear combination of yields. However, we find that estimation of a standard affine three-factor model results in a variance state variable that, while instrumental in explaining the shape of the yield curve, is essentially unrelated to GARCH estimates of the quadratic variation of the spot rate process or to implied variances from options. We then investigate four-factor affine models. Of the models tested, only the model that exhibits 'unspanned stochastic volatility' (USV) generates both realistic short rate volatility estimates and a good cross-sectional fit. Our findings suggest that short rate volatility cannot be extracted from the cross-section of bond prices. In particular, short rate volatility and convexity are only weakly correlated.

1. Introduction

This paper investigates the relation between interest rate volatility and the cross section of bond yields. It is well-established that at least three factors are needed to capture bond yield dynamics: Litterman and Scheinkman (1991) interpret them as level, slope, and curvature. It is also well-established that interest rate volatility is stochastic.[1] As such, this paper focuses on those three- and four-factor models of the term structure that capture stochastic volatility.

By imposing only the condition of no-arbitrage, it can be shown that state variables that drive changes in interest rate volatility generally play a 'dual role' in that they also drive changes in bond yields.[2] For example, theoretical considerations generally imply a strong link between changes in short-rate volatility and changes in curvature. Not surprisingly, this prediction is manifest in the specific models proposed in the literature. For example, affine models of the term structure predict that state variables driving interest rate volatility are also

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[1] See, for example, Fama (1976), Engle, Lilien, and Robins (1987), Brenner, Harjes, and Kroner (1996), and Han (2007).

[2] See, for example, Litterman, Scheinkman, and Weiss (1991), and Collin-Dufresne, Goldstein, and Jones (CGJ, 2008, p. 785), and Eq. (11) below.

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linear combinations of yields. As a special case, the 'preferred' $A_1(3)$ model of Dai and Singleton (DS, 2000) predicts that the variance of the spot rate is also a linear combination of the level, slope, and curvature of the yield curve.

The first goal of this paper is to investigate whether the variance state variable in the preferred $A_1(3)$ model can simultaneously satisfy its dual roles as the source of time-varying yield volatility and a factor in the yield curve. We choose this model because Duffee (2002) concludes that it offers the best forecasting performance among three-factor models with stochastic volatility, while DS find that it offers the best characterization of unconditional yield volatilities and a sufficiently flexible correlation structure.[3]

On the other hand, there is evidence that the $A_1(3)$ model is misspecified.[4] Therefore, a more general goal of the paper is to investigate the joint dynamics of level, slope, curvature, and volatility factors within a more flexible four-factor affine framework that does not impose at the outset a deterministic link between them.

We also investigate a subset of the affine class that displays 'unspanned stochastic volatility' (USV). These models impose strong parameter constraints in order to generate bond yields that are independent of some of the variables driving volatility. As such, these 'unspanned' variables do not play a dual role and thus are free to more accurately capture the time-series properties of interest rate volatility. This potential benefit is not without costs, however. First, the large number of parameter constraints may prove to be overly restrictive. For example, the $A_1(4)$ USV model has only 11 risk-neutral parameters that affect bond yields, compared to 22 for its unrestricted counterpart. Second, limiting a variance state variable to have only a time-series role means that such a model will be less able to explain cross-sectional yield patterns. For example, the $A_1(4)$ USV model has only three factors to capture the cross section of yields.

Prior empirical work on USV has focused almost exclusively on the spanning relation between interest rate derivatives and bond prices. For example, Collin-Dufresne and Goldstein (CDG, 2002) find changes in swap rates are only weakly correlated with returns on at-the-money cap straddles, while Heidari and Wu (2003) obtain similar results using implied volatilities from swaptions. Li and Zhao (2006) investigate quadratic models of the term structure and find them incapable of explaining the returns on caps of various maturities and strike prices. In contrast, Fan, Gupta, and Ritchken (2003) report that swaption returns are in fact well-spanned by yield changes, while both Bikbov and Chernov (2009) and Joslin (2007) report that the USV restrictions are strongly rejected when bond and option data are used.

Our paper differs from the previous work by focusing on the ability of USV models to explain bond prices themselves. Specifically, we are interested in determining whether USV models are able to simultaneously match both the cross-sectional and time-series properties of bond yields. While it seems clear that the USV model will match some aspects of the time series of volatility (i.e., conditional second moments of yields) relatively well, it is unclear how the restrictions imposed to generate USV will affect the model's ability to capture the cross section and time series of yields (i.e., conditional first moments).

We note that many standard econometric methods used to estimate affine models are unsuitable for investigating models exhibiting USV. Indeed, a consequence of imposing USV restrictions is that the one-to-one mapping assumed by Duffie and Kan (DK, 1996) between yields and factors does not hold. This implies that standard estimation techniques that rely on the 'invertibility' of the term structure (e.g., Chen and Scott, 1993; Pearson and Sun, 1994) with respect to the latent factors cannot be implemented. The Kalman filter-based approaches of Duan and Simonato (1999) and de Jong (2000) are also unsuitable for our purposes because USV restrictions make it impossible for a linear filter to properly update the distribution of the unknown volatility state variable. We therefore write term structure dynamics in a nonlinear state space form and estimate the parameters of the models using Bayesian Markov chain Monte Carlo (MCMC).

The results from estimating the unrestricted essentially affine three-factor model are striking. Most significantly, we obtain the 'self-inconsistent' result that the volatility factor extracted from this model (i.e., the 'term structure-implied volatility') is basically unrelated to volatilities estimated using rolling windows, GARCH volatilities, or implied volatilities from options. Furthermore, the strong in-sample fit of that model breaks down following the end of the estimation period, suggesting deep misspecification.

We interpret these findings as evidence that three-factor models cannot simultaneously describe the yield curve's level, slope, curvature, and volatility. That is, volatility is unable to play the dual role that such a model predicts it does. The estimation of such a model therefore presents a tradeoff between choosing volatility dynamics that are more consistent with one role or the other. For the data set we investigate, and with no parameter

[3] We note that recent empirical literature has reported that the affine class has trouble simultaneously fitting certain cross-sectional and time-series properties of the yield curve (Duffee, 2002; Dai and Singleton, 2002; Duarte, 2004). Indeed, these papers suggest that a more flexible risk premium structure is used to reconcile the time-variation in conditional variances and the forecasting power of the slope of the term structure. Further, Cochrane and Piazzesi (2005) find a state variable that drives risk premia but that is separate from the three factors affecting yield curve shape. The tradeoff we uncover here involves second-order moments, and thus is independent of the risk premia structure. Note that since the volatility structure is invariant under transformation from the historical measure to the risk-neutral measure, proposing a more general risk premia specification will not overcome this problem as it did in Duffee (2002).

[4] For example, Collin-Dufresne, Goldstein, and Jones (2008) propose a representation of affine models in terms of economically meaningful state variables (such as level, slope, and curvature) that can be estimated model-independently. They argue that if a model is well-specified, then state variables 'inverted' from prices using standard econometric techniques should be closely related to the model-independent estimates. They find for the $A_1(3)$ model that there are substantial deviations between the two when volatility is used as an observable state variable.