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Analyzing interest rate risk: Stochastic volatility in the term structure of government bond yields ☆

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ABSTRACT

We propose a Nelson–Siegel type interest rate term structure model where the underlying yield factors follow autoregressive processes with stochastic volatility. The factor volatilities parsimoniously capture risk inherent to the term structure and are associated with the time-varying uncertainty of the yield curve's level, slope and curvature. Estimating the model based on US government bond yields applying Markov chain Monte Carlo techniques we find that the factor volatilities follow highly persistent processes. We show that yield factors and factor volatilities are closely related to macroeconomic state variables as well as the conditional variances thereof.

1. Introduction

Much research in financial economics has been devoted to the modeling and forecasting of interest rates and the term structure thereof. However, only a few approaches explicitly account for time-varying interest rate risk. A potential reason is the typically high (cross-sectional) dimension of the term structure whose multivariate volatility is cumbersome to model. Consequently, most studies focus on the riskiness of (selective) interest rates for given maturities or study aggregated volatility measures based on a common component as recently suggested by Koopman et al. (2010) or based on bond market portfolios in the spirit of Engle et al. (1990) and Engle and Ng (1993).

In this paper, we propose capturing the riskiness inherent to the term structure of interest rates by an extended Nelson–Siegel (1987) term structure model where the underlying yield curve factors reveal stochastic volatility. We see this approach as a parsimonious alternative to a model where the individual time series of yields themselves reveal (high-dimensional) time-varying volatility. Modeling stochastic volatility directly in the Nelson–Siegel factors reduces the dimension of the stochastic volatility process to three and allows capturing time-varying uncertainty associated with the yield curve's level, slope and curvature. Accordingly, the so-called 'level volatility' reflects the volatility with respect to the overall level of yields, whereas the 'slope volatility' captures the time-varying riskiness in the spread between short-term and long-term yields. Correspondingly, the 'curvature volatility' is associated with the risk due to changes in the term structure curvature.

This paper contributes to the recent empirical literature on the modeling of interest rate dynamics. Classical equilibrium models, such as, e.g., Vasicek (1977), Cox et al. (1985), Duffie and Kan (1996), Dai and Singleton (2002) or Duffee (2002) and no-arbitrage approaches in the line of, e.g., Hull and White (1990) or Heath et al. (1992) describe the evolution of short rates in terms of underlying risk factors. They typically use affine structures which allow constructing the expected yields at other maturities given assumptions about the underlying dynamics of risk factors and risk

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Table 1
MCMC estimation results for dynamic Nelson–Siegel models. Based on monthly observations of unsmoothed U.S. Fama–Bliss zero coupon yields from January 1964 to December 2003 with maturities of one to five years. 480 observations.

	μ^L	μ^S	μ^C	ϕ^L	ϕ^S	ϕ^C	h^L	h^S	h^C	λ
A. Model without volatility factors										
Mean	0.028	−0.053	−0.028	0.994	0.956	0.906	0.307	0.597	0.757	0.055
SD	0.014	0.030	0.036	0.003	0.013	0.023	0.022	0.024	0.060	0.006
95% CI, lower	0.007	−0.116	−0.099	0.987	0.929	0.860	0.266	0.552	0.637	0.046
95% CI, upper	0.062	−0.001	0.042	0.998	0.980	0.947	0.351	0.645	0.871	0.069
Z-score	−0.171	1.198	1.615	0.196	1.352	−0.507	−1.737	−0.381	1.418	0.172

	μ^L	μ^S	μ^C	ϕ^L	ϕ^S	ϕ^C	μ^L_h	μ^S_h	μ^C_h	ϕ^L_h	ϕ^S_h	ϕ^C_h	$(\sigma^L_h)^2$	$(\sigma^S_h)^2$	$(\sigma^C_h)^2$	λ
B. Model with stochastic volatility factors																
Mean	0.023	−0.008	−0.025	0.994	0.981	0.879	−0.082	−0.083	−0.038	0.977	0.964	0.933	0.246	0.343	0.269	0.045
SD	0.011	0.008	0.034	0.002	0.006	0.024	0.053	0.037	0.023	0.016	0.017	0.034	0.078	0.070	0.083	0.002
95% CI, lower	0.005	−0.028	−0.092	0.988	0.966	0.830	−0.214	−0.169	−0.096	0.936	0.923	0.851	0.134	0.225	0.042	0.042
95% CI, upper	0.049	0.004	0.042	0.998	0.993	0.925	−0.017	−0.024	−0.005	0.996	0.992	0.987	0.439	0.498	0.048	0.048
Z-score	−1.647	−0.902	−1.085	1.615	0.878	1.463	−0.594	0.772	−1.196	−0.486	0.532	−1.383	0.378	−0.493	0.454	0.568

"95% CI" denotes the 95% credibility interval of the posterior distribution. The *Z*-score statistic is the Geweke (1992) test statistic for the convergence of MCMC samples, see Appendix A.

premia. These models – typically combined with stochastic volatility processes (e.g., Heston, 1993) – are workhorses for the pricing of bonds and interest rate derivatives. However, though these approaches are powerful in explaining the term structure across different maturities at a single point in time, they are not very successful in forecasting interest rates and the term structure thereof.

In fact, in both empirical research as well as financial practice, the latter task is dominantly addressed using factor models. A well-known approach in this area is the Nelson and Siegel (1987) exponential components framework which is neither an equilibrium nor a no-arbitrage model but can be heuristically motivated by the expectations hypothesis of the term structure. In this setting, the term structure is captured by three factors which are associated with the yield curve's level, slope and curvature. In a related approach, Litterman and Scheinkman, 1991 propose such factors as the first three principal components based on the bond return covariance matrix. Diebold and Li (2006) propose a simple dynamic implementation of the Nelson and Siegel (1987) model and employ it to model and to predict the yield curve. Diebold et al. (2006) extend this approach by including macroeconomic variables while Koopman et al. (2010) allow for time-varying loadings and include a common volatility component.

Motivated by the lacking empirical evidence on the role of term structure volatility, we aim at filling this gap in the literature and address the following three major research questions: (i) To which extent do yield curve factors reveal time-varying volatility? (ii) Are factor volatilities correlated with macroeconomic fundamentals? (iii) Are there dynamic interdependencies between macroeconomic variables, their conditional variances and term structure risk?

We represent the Nelson–Siegel model in a state space form, where both the (unobservable) yield factors and their stochastic volatility processes are treated as latent factors following autoregressive processes. We estimate the model using Markov chain Monte Carlo (MCMC) methods based on monthly unsmoothed Fama–Bliss zero yields from 1964 to 2003. In a second step, we use the estimated yield curve factors and volatility factors as components of a VAR model including macroeconomic variables, such as capacity utilization, industrial production, inflation, unemployment, GDP growth, as well as the federal funds rate, among others.

Based on our empirical study, we can summarize the following main findings: (i) We find strong evidence for time-varying term structure risk with the yield curve volatilities following persistent dynamics. (ii) Both yield factors and factor volatilities are strongly correlated with macroeconomic fundamentals, such as capacity utilization, inflation, unemployment rates, federal funds rates and GDP growth. (iii) Prediction error variance decompositions show evidence for significant long-run effects of macroeconomic state variables and their conditional variances on term structure movements and associated interest rate risk. Finally, in a forecasting exercise, we illustrate the effect of stochastic volatility in yield factors on the precision of yield forecasts.

The remainder of the paper is structured as follows. In Section 2, we describe the dynamic Nelson and Siegel (1987) model as put forward by Diebold and Li (2006) and discuss the proposed extension allowing for stochastic volatility processes in the yield factors. Section 3 presents the data and illustrates the estimation of the model using MCMC techniques. In Section 4, we investigate the dynamic interdependencies between yield factors, factor volatilities and macroeconomic variables. Section 5 shows the model's in-sample and out-of-sample forecasting ability. Finally, Section 6 concludes.

2. A dynamic Nelson–Siegel model with stochastic volatility

Let $p_t^{(n)}$ denote the log price of an n-year zero-coupon bond at time t with $t = 1, \ldots, T$ denoting monthly periods and $n = 1, \ldots, N$ denoting the maturities. Then, the yearly log yield of an n-year bond is given by $y_t^{(n)} := -\frac{1}{n} p_t^{(n)}$. The 1-year forward rate at time t for loans between time $t + 12(n-1)$ and $t + 12n$ is given by $f_t^{(n)} := p_t^{(n-1)} - p_t^{(n)} = n y_t^{(n)} - (n-1) y_t^{(n-1)}$. In the following we focus on 1-year returns observed on a monthly basis. Then, the log holding-period return from buying an n-year bond at time $t - 12$ and selling it as an $(n-1)$-year bond at time t is defined by $r_t^{(n)} := p_t^{(n-1)} - p_{t-12}^{(n)}$. Correspondingly, we define excess log returns as $z_t^{(n)} := r_t^{(n)} - y_{t-12}^{(1)}$.

Nelson and Siegel (1987) propose modeling the forward rate curve in terms of a constant plus a Laguerre polynomial function as given by

$$f_t^{(n)} = \beta_{1t} + \beta_{2t} e^{-\lambda_t n} + \beta_{3t} \lambda_t n e^{-\lambda_t n}. \quad (1)$$

Small (large) values of λ_t produce slow (fast) decays and better fit the curve at long (short) maturities. Though the Nelson–Siegel model is neither an equilibrium model nor a no-arbitrage model it can be still heuristically motivated by the expectations hypothesis of interest rates. If spot rates are generated by an underlying differential equation, then forward rates are the solutions thereof. Accordingly, Nelson and Siegel (1987) motivate 1 as an approximation to the solution of a second-order differential equation for