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# Econometric analysis of present value models when the discount factor is near one

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ABSTRACT

This paper develops asymptotic econometric theory to help understand data generated by a present value model with a discount factor near one. A leading application is to exchange rate models. A key assumption of the asymptotic theory is that the discount factor approaches one as the sample size grows. The finite sample approximation implied by the asymptotic theory is quantitatively congruent with the modest departures from random walk behavior that are typically found and with imprecise estimation of a well-studied regression relating spot and forward exchange rates.



## 1. Introduction

This paper develops and evaluates, via simulations and analytical calculations, asymptotic econometric theory intended to better explain the behavior of data generated by a present value model with a discount factor near one. A leading area of application is floating exchange rates. The research is motivated by two stylized facts that are well known in the asset pricing literature.

The first stylized fact is that it is difficult to document predictability in changes in asset prices such as floating exchange rates or equity prices, and that the bulk of the modest evidence we have of predictability comes from cross-correlations of asset price changes with other variables rather than univariate autocorrelations of asset price changes. Random walk like behavior in asset prices is *not* necessarily an implication of an efficient markets model such as the classic one exposited by Samuelson (1965). For example, in the exchange rate context, that efficient markets model, which assumes risk neutrality, requires that forward rates be unbiased and efficient predictors of spot rates. Yet forward rates are dominated by lagged spot rates as predictors of exchange rates. The classic reference on predicting exchange rates with forward rates and other variables is Meese and Rogoff (1983); recent updates are Engel et al. (2007) and Molodtsova and Papell (2009). Difficulties in prediction perhaps are less striking for equities than for exchange rates but are still notable; see Campbell and Thompson (2008) and Goyal and Welch (2008) for recent studies.

The second stylized fact pertains to exchange rates but not stock prices. It concerns the pattern of signs in regressions of levels or changes in exchange rates on forward rates on forward premia. (Here and throughout, exchange rates and forward rates are measured in logs.) In particular, the regression of the exchange rate change on the previous period's forward premium (i.e., the difference between the previous period's forward and actual exchange rates) yields a coefficient estimate that is sensitive to sample and to currency, but with tendency to be less than one. To

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**Table 1**
Forward-spot regressions, US dollar.

| | Canada | France | Germany | Italy | Japan | UK |
|---|---|---|---|---|---|---|
| (1) $\hat{\beta}_r$ | −0.204 | 0.486 | −0.767 | 1.697 | −2.703 | −1.433 |
| S.e. | (0.774) | (0.864) | (0.818) | (0.806) | (0.918) | (0.925) |
| (2) $\hat{\beta}_f$ | 0.982 | 0.927 | 0.976 | 0.939 | 0.986 | 0.904 |
| S.e. | (0.025) | (0.034) | (0.027) | (0.030) | (0.025) | (0.036) |
| (3) $\hat{\beta}_m$ | 1.159 | 1.149 | 1.054 | 1.239 | 1.016 | 1.144 |
| S.e. | (0.089) | (0.115) | (0.092) | (0.108) | (0.094) | (0.092) |
| Sample period | 79:2–11:1 | 79:2–98:4 | 79:2–11:1 | 79:2–98:4 | 79:2–11:1 | 79:2–11:1 |

Notes:
1. Data are quarterly. Three-month interest rates and end of quarter bilateral US dollar exchange rates 1979–2000 were obtained from Chinn and Meredith (2004), extended to 2001–2011 using LIBOR rates obtained from economagic.com and Bloomberg and end of quarter exchange rates obtained from International Financial Statistics. Forward rates were constructed using covered interest parity.
2. Let $y_t$ be the log of the exchange rate, $f_t$ the log of the three-month-ahead forward rate. Least squares estimates and standard errors of the following regressions are reported: $\Delta y_t = \text{const.} + \hat{\beta}_r(f_{t-1} - y_{t-1}) + \text{residual}$; $y_t = \text{const.} + \hat{\beta}_f f_{t-1} + \text{residual}$; $y_t - y_{t-2} = \text{const.} + \hat{\beta}_m(f_{t-1} - y_{t-2}) + \text{residual}$.

illustrate, consider Table 1, which reports three related regressions using quarterly bilateral US dollar exchange rates against the other six G7 countries. The first line of the table reports estimates of the regression of the exchange rate change on the previous quarter's forward premium. Five of the six estimates are below one, and four of the six are negative; the estimates range from −2.703 to 1.697. A value of one would obtain if forward rates were unbiased predictors of spot rates. This value is reflected in the second line of the table, which reports estimates of the regression of the level of the exchange rate on the level of the previous quarter's forward rate.[1] The coefficient estimates are near one. Since the first regression is a transformation of the second regression obtained by subtracting the lag of the exchange rate from the left-hand and right-hand sides, a coefficient of one would still obtain, at least in population, if the forward rate were an unbiased predictor of the spot rate. (The third regression will be discussed below.) One class of explanations for a value less than one in regressions such as those reported in the first line of Table 1 turns on time varying risk premia. Hansen and Hodrick (1983) and Fama (1984) are early papers on this topic; Verdelhan (2010) is a recent contribution. Other explanations for a value less than one turn on expectational or informational biases (e.g., Frankel and Froot, 1987, Bacchetta and van Wincoop, 2006) or on liquidity, transaction or portfolio costs (e.g., Baccchetta and van Wincoop, 2010).

To my knowledge, a systematic econometric theory to rationalize the first stylized fact – that is, to rationalize our findings of modest evidence against the random walk – has not been proposed. Among the possible econometric explanations for the second stylized fact is that the regression produces biased estimates. Indeed, the fact that the regression cannot be reliably used to predict exchange rate changes (the first stylized fact) suggests that the point estimates should not be taken at face value. Maynard and Phillips (2001), building on Baillie and Bollerslev (1994, 2000), develop a theory in which fractional integration of the forward premium leads to bias in the point estimate, even asymptotically. A number of authors have argued that, with samples of size available in practice, the regression produces estimates that are biased downward relative to ones predicted by asymptotic theory. Evans and Lewis (1995) present analytical arguments for bias that turn on small-probability events not being realized in the samples that we observe. They and other authors have found downward biases in simulations.

This paper offers econometric theory to help rationalize both stylized facts. Its starting point is Engel and West (2005). That paper shows that, in present value models with constant discount factors, random walk like behavior is expected in the population as the discount factor approaches unity. The present paper develops econometric theory motivated by that analytical result. I derive an asymptotic approximation that, given a DGP, allows one to determine how close to a random walk an asset price will behave in a finite sample: under this approximation, hypothesis tests of zero predictability of asset price changes have size greater than the nominal size that would apply for a pure random walk, but less than the unit size that would apply under the usual asymptotics. In congruence with the first stylized fact, calibration of exchange rate and stock price data sets implies size barely above nominal size for univariate autocorrelations, and modestly above nominal size for tests of cross-correlation.

This econometric theory also leads to a reinterpretation of the regression that is at the heart of the second stylized fact (a regression that, incidentally, is not discussed in Engel and West, 2005). According to the theory developed here, the regression of the exchange rate change on the previous period's forward premium is inconsistent: even if the forward rate is the expectation of the exchange rate, and even with large sample sizes, estimates of the slope coefficient will not have high probability of being clustered near one. Hence relatively little weight should be put on the estimates of this regression. Finally, the theory suggests that a closely related regression – that of the two-period change in the exchange rate on the difference between the previous period's forward rate and the two-period lagged exchange rate – should produce a coefficient of one. Some of the explanations of the second stylized fact given above imply that this coefficient will be less than one. A tendency of this regression to yield a coefficient near one has not achieved the status of a stylized fact. But this indeed may be the case, as is indicated by the estimates in the third line of Table 1 (which range from 1.054 to 1.239) and by the similar values reported for a different sample by McCallum (1994).

While this paper is motivated by results in the stock price and especially the exchange rate literature, the analytical results are derived for a general present value model. Hence they may be of interest in interpretation of results for other asset prices. For that reason, much of the exposition below refers to "the asset price".

It should be noted that the paper assumes throughout that risk premia are constant (a zero risk premium is a special case), and abstracts as well from possible contributions from informational biases or liquidity or transactions costs. On the roles of such factors for exchange rates, see the references above; for stock returns, see, e.g., Campbell et al. (1997) or Barberis and Thaler (2003). Whatever those roles may be for asset prices and returns, my aim in the present paper is to supply a relatively clean analysis of behavior

[1] Since the exchange and forward rate arguably are $I(1)$ variables, the standard errors should be interpreted with caution. Another reason for caution in interpreting all the point estimates in the table is that, as stated in the notes to the table, forward rates were constructed using covered interest parity, and thus likely depart slightly from measured forward rates (Bhar et al., 2004). I include this table not to present new empirical results, but to focus the econometric discussion.