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Technical note: Equivalence of different profitability criteria with the net present value

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ABSTRACT

Net present value has long been regarded in academic circles as the best criterion for project appraisal; however, several alternative, complementary methods remain popular with practitioners. This paper demonstrates that, if properly applied, several of these standard criteria – such as net final value, internal rate of return, benefit–cost ratio, profitability index, equivalent annuity, discounted payback period and average payback period – lead to the same investment decision as net present value. Moreover, the paper proves that when choosing between two mutually exclusive projects, the application of these criteria to the difference project provides the same ranking as net present value. Therefore, although net present value is regarded as a superior investment criterion, any of these popular criteria, properly applied, serve as well.

1. Introduction

Several authors expose the merits and limitations of the different profitability criteria applied to project appraisal (e.g., Remer and Nieto, 1995a,1995b; Karibskii et al., 2003; and Godinho et al., 2004). The net present value (NPV) criterion is generally considered superior and is widely used, especially by large firms (Graham and Harvey, 2001; Brounen et al., 2004). It is even employed as a basis for the incorporation of new variables such as uncertainty in project valuation (De Reyck et al., 2008). Meanwhile, for the other criteria, such as the very popular internal rate of return (IRR), the main criticism is their lack of equivalence with NPV (e.g., Ross, 1995; Oehmke, 2000; Magni, 2010; and Chiang et al., 2010). However, in spite of persistent criticism in academic circles, IRR as well as other alternative and complementary criteria (e.g., payback period, PB, or benefit–cost ratio, BCR) are still very popular among practitioners (Remer et al., 1993; Ryan and Ryan, 2002). It is therefore essential to demonstrate whether the application of these widely used criteria is consistent with NPV, the theoretically correct criterion.

One of the first problems leading to the rejection of a criterion is that it may give a different response to NPV for the desirability of a project. Several authors (e.g., Hirshleifer, 1958; Rosen, 2008) highlight the issue of potential multiple IRRs for a given project, pointing out that in such a case, this criterion does not provide an adequate solution. Some authors have tried to address this question by establishing an application rule for the IRR criterion in order to obtain an equivalent evaluation to that obtained with NPV. Hazen (2003,2009) states that in the case of multiple (even complex) internal rates of return, each can meaningfully be interpreted as a rate of return on its own underlying investment stream (sequence of capitals periodically invested in the project). Irrespective of which rate is used, once the underlying investment stream is identified as net investment or net borrowing, the decision is consistent with NPV. Pasqual et al. (2001, 2005) demonstrate that the alleged conflict between NPV and IRR on the profitability of a project can easily be overcome by considering the characteristics of the NPV function and gives appropriate definition of what investments and loans (borrowing decisions) are. The authors show that all the real IRRs make sense from an economic standpoint and that when there is at least one real IRR, both IRR and NPV lead to the same recommendations (see Section 2.3 below). Their method does not require the transformation of cash flow streams. Using the same conceptual framework, Hartman and Schafrik (2004) independently derive the same basic results: if there is at least one real IRR, NPV and IRR always coincide in their recommendations on the desirability of a project. Later, Zhang (2005) examines the same problem and proposes a method that focuses on the parity of the number of real IRRs that are greater than the cost of capital, so that the decision rule is consistent with NPV.

More recently, Magni (2010), who provides a comprehensive review of the relevant literature, suggests the use of the average

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internal rate of return (AIRR) as an alternative to IRR. The author defines AIRR as an average of one-period return rates derived from investment streams that are freely chosen by the analyst. The criterion then compares the AIRR with the market rate. The AIRR method is fairly successful at obtaining a criterion expressed as a rate of return that is consistent with NPV. Additionally, it overcomes the problem of the non-existence of real IRRs by providing a real-valued measure that can also be applied to both gifts and losses. Pierru (2010) provides an interpretation of complex rates of return in project appraisal. He states that a series of real rates of return can be associated with any complex rate of return. His proposed alternative makes it possible to discount at a single (but complex) rate the cash flow of an investment involving the joint production of two outputs, the markets of which have different risks. Finally, Osborne (2010) considers all the different rates (real and complex) and shows that the NPV per dollar invested is composed of all multiplicative mark-ups of every IRR over the cost of capital. Osborne (2010, p. 237) contends that "every IRR is equally important, because NPV is composed of them all". The pitfall of different IRRs is not viewed as a problem by Osborne as all of them provide information contained in the NPV function. Also, the ranking provided by considering all possible IRRs would be identical to that availed by the NPV per dollar invested. According to Osborne, the IRR is not a criterion per se but rather a component of the NPV criterion that provides partial information.

This paper contributes to the existing literature by extending to other criteria the above cited contributions on the equivalence between NPV and real IRRs for deciding on project desirability. In short, we will demonstrate the equivalence with NPV for other fairly popular criteria: net final value (NFV), benefit–cost ratio (BCR), profitability index (PI), equivalent annuity (EA), discounted payback period (PB) and average payback (APB) (a new criterion with broader scope of application than PB). We find, given that these are widely used methods, that it is of major relevance to determine whether their application, whenever possible, is equivalent to applying NPV.

A second problem that can cause a criterion to be considered inappropriate is the possibility of a ranking of mutually exclusive projects different from the one suggested by the NPV criterion. This is the case with IRR and BCR, among others. The solution to this problem is intuitively offered in, inter alia, Gittinger (1984), Brent (1998), Newnan et al. (2011), Zerbe and Bellas (2006) and Brealey et al. (2011). These authors claim that the appropriate way of choosing between exclusive projects with the IRR criterion is to apply it to the difference project. Unfortunately, none of them provides any formal demonstration and they limit their recommendation to the IRR criterion. An exception is Magni (2011), who shows that the application of AIRR to the difference project leads to a ranking consistent with NPV. In this paper, we demonstrate that the application of IRR to the difference project is always consistent with NPV and we extend this result to all the criteria that are equivalent to NPV. So, it would also be appropriate to use them to rank mutually exclusive projects.

2. Criteria to assess the desirability of a project

A project $P(X_t, r_t)$ consists of flows X_t, $t=M, \ldots, M+T$, where M is the period at which the project is implemented, T its duration and r_t the discount rate. Unless otherwise stated, it is assumed that $r_t=r$ and $M=0$.

The difference project $(P-Q)$, P with flows X_t, $t=M, \ldots, M+T$ and Q with flows $Y_{t'}$, $t'=M', \ldots, M'+T'$, is characterised by the flows $(X_t-Y_{t'})$ and a time span formed by the union of the two time spans: $[\min\{M, M'\}; \max\{M+T; M'+T'\}]$. The difference project quantifies the impact of implementing project P rather than Q.

In order to apply the different criteria, it is convenient to define four types of project – investment, loan, gift, and loss – , as done below. This classification is also convenient for a better understanding of the economic meaning of the application of the different criteria:

Investment: If there are strictly positive and negative flows, the project behaves as an investment over the interval $[r^a, r^b]$, $a \le b$, if $\partial NPV/\partial r < 0$ over the interval $[r^a, r^b]$.

Loan: If there are strictly positive and negative flows, the project behaves as a loan (borrowing decision) over the interval $[r^a, r^b]$, $a \le b$, if $\partial NPV/\partial r > 0$ over the interval $[r^a, r^b]$.

Gift: All the flows are non-negative and at least one is strictly positive.

Loss: All the flows are non-positive and at least one is strictly negative.

2.1. Net present value (NPV)

The NPV function measures the increase in net wealth at the moment that would be equivalent to the implementation of the project.

$$NPV(X_t; r) = X_0 + X_1(1+r)^{-1} + X_2(1+r)^{-2} + \ldots + X_{T-1}(1+r)^{-(T-1)} + X_T(1+r)^{-T} \tag{1}$$

where $r \neq -1$. In what follows we will only consider the case where $r > -1$.

NPV is applicable to any type of project. Project P is accepted if and only if:

$$NPV(P) \ge 0 \tag{2}$$

and the higher NPV the better for any type of project.

For two mutually exclusive projects P and Q:

$$P > Q \Leftrightarrow NPV(P) \ge NPV(Q) \tag{3}$$

$$\Leftrightarrow NPV(P-Q) \ge 0 \tag{4}$$

2.2. Net final value (NFV)

The NFV function measures the increase in net wealth in the final period T that would be equivalent to the implementation of the project.

$$NFV(X_t;\ r) = X_0(1+r)^T + X_1(1+r)^{T-1} + \ldots + X_{T-1}(1+r) + X_T, \forall r \tag{5}$$

NFV is applicable to any type of project

Project P is accepted if and only if:

$$NFV(P) \ge 0 \tag{6}$$

and the higher NFV the better for any type of project.

NFV is consistent with NPV:

$$NFV(P) = (1+r)^T \cdot NPV(P) \tag{7}$$

$$(1+r)^T \cdot NPV(P) \ge 0 \Leftrightarrow NPV(P) \ge 0, \forall r > -1 \tag{8}$$

$$NFV(P) \ge 0 \Leftrightarrow NPV(P) \ge 0, \forall r > -1 \tag{9}$$

2.3. Internal rate of return (IRR)

IRR measures the increase in capital in relative terms and determines the growth rate of gross capital per period.

$$\text{IRR is all } r_j^* \text{ such that } NPV(X_t, r_j^*) = 0,\ j = 0, 1, \ldots, J \tag{10}$$

In what follows we will only refer to real IRRs greater than -1. The conventional IRR is not applicable to gifts or losses. It is applicable to investments and loans, provided that there is at least one real IRR. Neither the existence nor the uniqueness of IRR is