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A number-dependent replacement policy for a system with continuous preventive maintenance and random lead times

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ABSTRACT

This paper considers a number-dependent replacement policy for a system with two failure types that is replaced at the nth type I (minor) failure or the first type II (catastrophic) failure, whichever occurs first. Repair or replacement times are instantaneous but spare/replacement unit delivery lead times are random. Type I failures are repaired at zero cost since preventive maintenance is performed continuously. Type II failures, however, require costly system replacement. A model is developed for the average cost per unit time based on the stochastic behavior of the system and replacement, storage, and downtime costs. The cost-minimizing policy is derived and discussed. We show that the optimal number of type I failures triggering replacement is unique under certain conditions. A numerical example is presented and a sensitivity analysis is performed.

1. Introduction

Avoiding system failure is paramount when such an event is costly and/or dangerous. For these situations, reliability theory studies maintenance policies that manage operating costs and risks of a catastrophic breakdown. This study seeks to determine a replacement policy when such a system is subject to stochastic failures with each failure weakening the system and making operation more expensive.

Preventive replacement policies for systems that are subject to stochastic failures have been treated extensively in the literature. Barlow and Proschan [1] proposed the traditional "age-replacement policy" approach: replace the system at failure or at age T, whichever comes first. Several extensions of this policy have been investigated; see, for example, Bai and Yun [2], Cleroux [3], Nakagawa and Kowada [4], Block et al. [5], Berg et al. [6], Sheu et al. [7], and Chien and Sheu [8]. Boland and Proschan [9] considered the case of periodic replacement or overhaul at times $T, 2T, \ldots,$ for some $T > 0$ and minimal repairs at interim failures. This model was extended by Nakagawa [10,11], Boland [12], Nguyen and Murthy [13], A-Hameed [14], Ait Kadi and Cleroux [15], Sheu [16,17], among others. Further, Makabe and Morimura [18–20] proposed a new replacement model where a system is replaced at the nth failure and discussed the optimum policy. This model has been generalized by Morimura [21], Park [22,23], Nakagawa [10,24], Nakagawa and Kowada [4], Sheu [16], Sheu and Griffith [25], Sheu et al. [7], and Chien et al. [26].

Most policies studied in the literature assume that at any time there is an unlimited supply of units available for replacement; typically spare units are assumed to be inexpensive and bulk purchased under a stocking policy. However, this

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assumption is often unrealistic. When spare units are expensive and/or storage is costly, an enterprise may store few if any spares. In addition, since downtime can be costly when delivery lead times are lengthy, it is important to consider spare unit costs, delivery lead times, and storage costs. Nakagawa and Osaki [27] discussed an age-replacement policy with random lead times. Osaki and Yamada [28] discussed an age-replacement policy with random lead times and costs for storage and system downtime. Sheu and Griffith [29] and Sheu and Chien [30] considered a generalized age-replacement policy with age-dependent minimal repairs and random lead times. Chien and Chen [31] further developed an optimal spare ordering policy under random lead times for products under a rebate warranty.

In this paper, a number-dependent replacement model is presented for a costly system that is replaced at the nth type I (minor) failure or the first type II (catastrophic) failure, whichever occurs first, under continuous preventive maintenance and random lead times. We assume continuous preventive maintenance since the performance of almost all systems deteriorates with age and is negatively impacted by minor failures in terms of operating costs and output quality. Consequently, all minor failures are detected repaired instantaneously. Our objective is to derive the expected cost rate for this system and to characterize the cost-minimizing number of minor failures that trigger system replacement.

It is worth pointing out that there may be many failure modes in general production systems. In this study, however, for ease of model formulation and to facilitate comparison with alternative policies, we consider only two failure types. A type I failure is minor and can be repaired immediately, whereas a type II failure is catastrophic and requires system replacement. The relatively benign impact of a type I error comes at the cost of continuous preventive maintenance. As described in Pham and Wang [32], preventive maintenance is any maintenance that occurs when the system is operating. According to a standard definition in the reliability literature, preventive maintenance means "all actions performed in an attempt to retain an item in specified condition by providing systematic inspection, detection, and prevention of incipient failures" [33]. In this paper, the idea of continuous preventive maintenance can be regarded as meaning that a dedicated maintenance service facility is active at all times. This assumption is consistent with the assumption that the system under study is large, complex, and/or expensive and that downtime is catastrophic. As a consequence of this assumption, all type I failures are not only detected instantaneously and but also repaired without additional costs.

Many authors deal with the repair-replacement problem by assuming that system managers examine the state of a failed system and take a corresponding corrective action with immediate results. That is, based on the underlying condition of the system, minimal repairs are performed or the system is replaced with a new one instantaneously. This is called "condition-based maintenance"; see Aven [34,35], Aven and Gaarder [36], Bergman [37], Finkelstein [38], and Aven and Jensen [39], among others. However, this maintenance scheme is not continuous since only failures prompt inspection. It is not preventive since corrective actions are taken only in response to failure. And it usually does not incorporate lead times for replacement. Assuming continuous preventive maintenance is a novel feature of the model studied in this paper and a practical step towards modelling the reality of such systems.

The rest of this paper is organized as follows. Assumed system characteristics are described in Section 2. Primary formulation and model development are presented in Sections 3 and 4. Section 5 derives the total expected long-run cost per unit time, and Theorem 1 presents the general optimization results. In Section 6, a numerical example is given for illustration. Conclusions are drawn in Section 7.

2. System description

We consider a one-unit system under preventive maintenance with the following six basic characteristics.

1. System failure at age t can be one of two failure types. A type I failure occurs with probability $q(t)$ and is repaired immediately. A type II failure occurs with probability $p(t) = 1 - q(t)$ and requires system replacement. If the nth type I failure occurs before the first type II failure, the system is replaced; such a replacement can be regarded as a preventive replacement.
2. Preventive maintenance is performed continuously during the operation of the system, so that all system failures are detected instantaneously and type I failures are repaired immediately. A type II failure prompts immediate replacement if a spare is available; otherwise, replacement immediately follows any remaining delivery lead time.
3. The system has a failure time distribution $F(t)$ with finite mean μ and has probability density function (pdf) $f(t)$. The associated failure or hazard rate is $r(t) = f(t)/\overline{F}(t)$ and the cumulative hazard is $R(t) = \int_0^t r(x)\mathrm{d}x$, which is related to the survival function $\overline{F}(t) = 1 - F(t)$ by $\overline{F}(t) = \exp\{-R(t)\}$. It is further assumed that the failure rate of the system, $r(t)$, is continuous, monotone, and unaffected by type I repairs.
4. The lead time L for the delivery of a replacement unit has distribution function $H(t)$, pdf $h(t)$, survival function $\overline{H}(t)$, and finite mean $E(L) = \mu_L$.
5. If the replacement unit is received before the nth type I or the first type II failure occurs, the replacement is made immediately upon either of these triggers. Otherwise, replacement must wait the length of the remaining lead time since the last replacement.
6. Operation of the original system begins at time 0, a replacement unit is ordered at time 0, and one additional spare/replacement unit is ordered at the time of any replacement.