Int. J. Production Economics 118 (2009) 361–366

Contents lists available at ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

# Opportunistic preventive maintenance scheduling for a multi-unit series system based on dynamic programming

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ARTICLE INFO

*Article history:*
Received 14 March 2006
Accepted 30 September 2008
Available online 22 October 2008

*Keywords:*
Preventive maintenance
Imperfect
Opportunistic
Dynamic
Scheduling

ABSTRACT

It is understood that for a multi-unit series system, whenever one of the units stops to perform a preventive maintenance (PM) action, the whole series system must be stopped. At that time PM opportunities arise for the other units in the system. This paper proposes an opportunistic PM scheduling algorithm for the multi-unit series system based on dynamic programming with the integration of the imperfect effect into maintenance actions. An optimal maintenance practice is determined by maximizing the short-term cumulative opportunistic maintenance cost savings for the whole system. Matlab is considered for the optimization which is based on numerical simulation. Numerical examples are given throughout to show how this approach works. Finally, a comparison between the proposed PM model and other models is given.



## 1. Introduction

It is understood that conducting proper maintenance is an effective way to keep manufacturing systems in good condition. Since Barlow and Hunter gave the minimal repair model in 1960 (Barlow and Hunter, 1960), a lot of efforts have been made on maintenance scheduling which play great roles in improving operational safety, minimizing maintenance costs, and reducing the frequency and severity of in-service system failures.

Traditionally, preventive maintenance (PM) is scheduled periodically based on a technician's experience and it often hold a same time interval for PM activities (Nakagawa, 1984; Canfield, 1986; Sheu et al., 1995). In practice however, PM activity is generally imperfect and it cannot restore the system to as good as new. Even though some degraded components are replaced during PM activities, the cumulative wear on adjacent components may go unnoticed and worsen the condition of the relative parts, and the system as a whole. Therefore, the age-T policy will give unavoidably decreasing reliabilities to the deteriorating systems with imperfect maintenance effect.

Fortunately, many imperfect PM models have been developed which pay much attention to the system reliability (Pham and Wang, 1996). In fact, under an imperfect circumstance, a system will be preventive maintained at a decreasing sequence of time intervals, which could be more practical since most systems need more frequent maintenance with increased usage and age.

On the other hand, with the increasing demand from industry, more and more research efforts are being directed toward multi-unit system modeling. In term of the different system configurations (i.e., series, parallel, k-out-of-n), different maintenance models for multi-unit systems have been proposed (Berg, 1978; Zheng and Fard, 1992; Pham and Wang, 2000; Jhang and Sheu, 2000; Marseguerra et al., 2002; Bris et al., 2003; Karin et al., 2007; Allaoui et al., 2008; Rachaniotis and Pappis, 2008). However, two issues still need to be addressed. First, the majority of these models that appear in the literature

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doi:10.1016/j.ijpe.2008.09.012

**Nomenclature**

| | |
|---|---|
| $i$ | ordinal of PM cycles |
| $j,k,l$ | symbol of unit, $j,k,l\in\{1,\dots,n\}$ |
| $n$ | number of units |
| $a_{ij}$ | age reduction factor for unit $j$ during PM cycle $i$ |
| $T$ | mission time |
| $h_{ij}(t)$ | system hazard rate function prior to the $i$th PM for unit $j$ |
| $T_{ij}$ | PM interval prior to the $i$th PM for unit $j$ |
| $R_j$ | reliability threshold for unit $j$ |
| $\tau_{pj}$ | duration of PM for unit $j$ |
| $c_{pj}$ | PM cost per unit time for unit $j$ |
| $c_{dj}$ | downtime cost during PM per unit time for unit $j$ |
| $C_{mj}$ | minimal repair cost for unit $j$ |
| $N_j$ | number of PM actions for unit $j$ during mission time $T$ |
| $CE_j$ | expected cost per unit time for unit $j$ in mission time $T$ |
| $m_j,\eta_j$ | parameters of Weibull distribution |
| $CostSave_{jl}$ | cost saving for advancing the PM action of unit $j$ with unit $l$ |
| $DC_{jl}$ | downtime cost saving for advancing the PM action of unit $j$ with unit $l$ |
| $MC_{jl}$ | maintenance cost saving for advancing the PM action of unit $j$ with unit $l$ |
| $PC_{jl}$ | penalty cost for advancing the PM action of unit $j$ with unit $l$ |
| $R_{jl}$ | reliability of unit $j$ when unit $j$ is maintained with unit $l$ |
| $T_{ijo}$ | original PM interval for unit $j$ prior to the $i$th PM |
| $T_{ijn}$ | new PM interval for unit $j$ prior to the $i$th PM because of advancing of PM |
| $h_{ijo}(t)$ | original hazard rate function for unit $j$ prior to the $i$th PM |
| $h_{ijn}(t)$ | new hazard rate function for unit $j$ prior to the $i$th PM because of advancing of PM |
| $\delta T_j$ | cumulative PM time shift for unit $j$ from the old PM schedule to the new one |
| $R_{initial}$ | unit's initial reliability at beginning of a decision cycle |
| $t_{k1}$ | time point when unit $k$ reaches its reliability threshold firstly during a decision cycle |
| $t_{k2}$ | time point when unit $k$ reaches its reliability threshold secondly during a decision cycle |
| $t_j,t_l$ | time point for unit $j$ or unit $l$ when it reaches its reliability threshold during a decision cycle |
| $OC_{jl}$ | OM (opportunistic maintenance) cost saving for unit $j$ to advance the PM action with unit $l$ |
| $G$ | combination of PM activities |
| $G^*$ | optimal combination of PM activities |
| $T^*_{Nj}$ | residual time since the last PM action for unit $j$ during mission time $T$ |

assume that conducting maintenance will always restore a system to an 'as good as new' condition (Berg, 1978; Zheng and Fard, 1992; Jhang and Sheu, 2000; Marseguerra et al., 2002; Bris et al., 2003; Karin et al., 2007; Allaoui et al., 2008; Rachaniotis and Pappis, 2008). In many cases though, this assumption has proven to be far from the truth. Second, many models always obtained the optimal solution by minimizing the long-run average maintenance cost (Berg, 1978; Zheng and Fard, 1992; Pham and Wang, 2000; Jhang and Sheu, 2000; Marseguerra et al., 2002). However, this is not practical because usually information is only available over a short term, especially under a multi-unit condition.

In this paper, a dynamic opportunistic PM scheduling algorithm for a multi-unit series system is proposed based on short-term optimization with the integration of imperfect effect into maintenance actions. Whenever one of the units reaches its reliability threshold, a PM action will be performed on that unit. At that time the whole system has to stop because of the serial form of the units and PM opportunities arise for the other system units. The optimal PM activities are determined by maximizing the short-term cumulative OM (opportunistic maintenance) cost savings of the whole system based on the set-up cost concept presented by Wildeman et al. (1997).

For a literature overview of the field of imperfect maintenance, we refer to the review article by Pham and Wang (1996). According to this article, there are eight main methods for modeling an imperfect PM action. One of them is the improvement factor method (Malik, 1979; Nakagawa, 1988; Martorell et al., 1999; Levitin and Lisnianski, 2000; Zhao, 2003; Tsai et al., 2004). If $T_{ij}$ and $h_{ij}(t)$ for $t\in(0,T_{ij})$, respectively, represent the PM interval and the hazard rate function of unit $j$ prior to the $i$th PM, the hazard rate function after the $i$th PM $h_{(i+1)j}(t)$ becomes $h_{ij}(t+a_{ij}T_{ij})$ for $t\in(0,T_{(i+1)j})$ where $0<a_{ij}<1$ is the age reduction factor due to the imperfect PM action. This implies that each imperfect PM changes the initial hazard rate value right after the PM to $h_{ij}(a_{ij}T_{ij})$, but not all the way to zero (not new). The improvement factor method is very useful in engineering because a maintenance decision is always in terms of the system hazard rate or other reliability measures. Therefore in this research, we consider the improvement factor method in imperfect PM modeling.

## 2. Imperfect PM modeling for single-unit systems

First, when considering a single-unit system, the main assumptions concerning the imperfect PM model are as follows:

(1) The unit undergoes relatively constant conditions involving stress, its environment and the maintenance in a given mission time $T$.
(2) Whenever the unit fails, a minimal repair is performed on that unit. Minimal repair can only recover the unit's function and it cannot change the unit's hazard