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Optimal preventive maintenance policy for leased equipment using failure rate reduction

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A B S T R A C T

This study proposes a maintenance scheme for leased equipment using failure rate reduction method and derives an optimal preventive maintenance (PM) policy that minimizes expected total cost. Under the proposed maintenance scheme, the lessor (equipment owner) rectifies failures with minimal repairs within the lease period, and the lessor may incur a penalty when repair time exceeds a time limit as specified in the lease contract. To reduce the expected total cost, the lessor may employ PM actions to decrease the number of possible failures. In this study, an efficient algorithm is developed to derive the optimal PM policy and a closed-form solution is obtained for the case where the lifetime distribution of the equipment is Weibull. The expected total cost using the optimal PM policy under the proposed maintenance scheme is then compared with the performance of other policies under various maintenance schemes through numerical examples.

1. Introduction

Most businesses require various types of equipment to manufacture their products or to provide service for customers. Due to rapid technological innovations, increased complexity of equipment, and the cost of professional technicians required to maintain equipment, it may not be economical for these businesses to own certain equipment. Therefore, there is a trend toward leasing instead of buying equipment (Glickman & Berger, 1976; Nisbet & Ward, 2001). For leased equipment, the maintenance of the equipment is usually specified in a lease contract provided by the lessor (equipment owner) to ensure that the equipment could fulfill its intended purpose (Barlow & Hunter, 1960). As a result, the equipment was bundled with maintenance and offered by the lessor under a leased contract (Martin, 1997; Murthy & Asgharizadeh, 1999).

In general, two types of maintenance actions are considered in a lease contract – corrective maintenance (CM) and preventive maintenance (PM). Corrective maintenance rectifies failed equipment back to its operational status, whereas PM improves the operational status of the leased equipment, thereby decreasing the likelihood of equipment failure. There is a vast literature dealing with maintenance policies (Barlow & Hunter, 1960; Chun, 1992; Glickman & Berger, 1976; Jack & Dagpunar, 1994; Jaturonnatee, Murthy, & Boondiskulchok, 2006; Murthy & Yeung, 1995; Nakagawa, 1981; Nguyen & Murthy, 1988; Pham & Wang, 1996; Pongpech & Murthy, 2006; Seo & Bai, 2004; Sheu, Lin, & Liao, 2006; Wang, 2002; Yeh & Chen, 2006; Yeh & Lo, 2001). In practice, minimal repair is the most commonly performed CM when restoring failed equipment (Nakagawa, 1981; Nakagawa & Kowada, 1983). Following minimal repair, the equipment is operational; however, the failure rate remains unchanged. When the time needed for minimal repair exceeds the limit specified in the lease contract, the lessor might incur a penalty since it may cause serious damage to the lessee (equipment user). Therefore, a lessor must undertake some remedial measures to avoid costs incurred by equipment failures.

Most lessors undertake PM to reduce the number of equipment failures within the lease period. Preventive maintenance is a tradeoff between PM costs and failure costs. Usually, as PM is planned, the cost of PM is less than the cost incurred when equipment fails. Numerous PM policies have been proposed and studied under various situations, such as perfect or imperfect maintenance (Brown & Proschan, 1983; Jack & Dagpunar, 1994; Jaturonnatee et al., 2006; Pham & Wang, 1996; Sheu et al., 2006), age reduction or failure rate reduction (Chan & Shaw, 1993; Jaturonnatee et al., 2006; Nakagawa, 1981; Pongpech & Murthy, 2006), and periodical or sequential maintenance (Chun, 1992; Jack & Dagpunar, 1994; Pongpech & Murthy, 2006; Seo & Bai, 2004; Yeh & Chen, 2006; Yeh & Lo, 2001).

Jaturonnatee et al. (2006) developed a sequential PM scheme using failure rate reduction that considers the number of PM actions, PM degree, and time epochs simultaneously. Their

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maintenance scheme is very general but not easy to implement in practice. For practical needs, Pongpech and Murthy (2006) reduced Jaturonnatee's scheme to a periodical PM scheme in which the PM actions are carried out periodically with various maintenance degrees. Since this maintenance scheme is a special case of Jaturonnatee's scheme, the resulting performance is not as good as Jaturonnatee's. This study proposes a maintenance scheme, in which preventive maintenance actions are taken to reduce the failure rate of the leased equipment by the fixed amount specified in the lease contract. We employ a different approach to simplify Jaturonnatee's scheme. Under our approach, the PM actions are performed sequentially with a fixed maintenance degree. As we will see later on, the performance of the proposed maintenance scheme is better than Pongpech's scheme and is close to that of Jaturonnatee's scheme.

The remainder of this paper is as follows. The mathematical model of the proposed maintenance scheme is developed in Section 2. In Section 3, the optimal PM policy is examined and an efficient algorithm is proposed for leased equipment with general lifetime distributions. In Section 4, the optimal PM policy is derived and a reduced algorithm is proposed for the Weibull lifetime distribution. The performance of PM is evaluated via numerical examples, the expected total cost is compared with two other maintenance schemes (Jaturonnatee et al., 2006; Pongpech & Murthy, 2006), and some practical applications are given in Section 5. Finally, conclusions are drawn in Section 6.

2. Mathematical formulation

Given that the failure rate of equipment, $h(t)$, is a strictly increasing function (degenerating equipment) over time t with $h(0) = 0$; within the lease period, failed equipment is repaired using minimal repair by the lessor with a fixed repair cost C_m. Following minimal repair, the equipment is operational; however, its failure rate remains the same as that just prior to failure. Assume that any minimal repair requires a random amount of repair time T_m, which follows a general cumulative distribution function G. Each failure incurs a fixed penalty cost C_n to the lessor. Furthermore, if the repair time exceeds a predetermined value τ, then there is a penalty C_τ per unit time to the lessor for the delay in restoring the equipment back to operational condition. That is, the total expected cost to the lessor at each failure is $C_m + C_n + C_\tau \int_\tau^\infty \overline{G}(t)\mathrm{d}t$.

To reduce the number of possible failures, the lessor may perform n PM actions within the lease period. After performing the ith PM action at time epoch t_i, the failure rate of the equipment is reduced by a fixed amount $\delta \geqslant 0$, where $0 < t_1 < t_2 < \cdots < t_n < L$. In practice, the cost of a PM action is a non-negative and non-decreasing function of the maintenance degree $\delta \geqslant 0$. In this paper, we consider the case where the PM cost function $C_{pm}(\delta)$ increases linearly with maintenance degree δ; those is, $C_{pm}(\delta) = a + b\delta$ where $a > 0$ and $b \geqslant 0$ are the fixed cost and the variable cost for each PM action, respectively. Furthermore, it is assumed that the time required for performing minimal repair and PM actions are both very short compared to the leased period and, hence, are negligible.

Without any PM actions, the failure process of equipment is a non-homogeneous Poisson process (NHPP) with intensity $h(t)$, since minimal repairs (Nakagawa, 1981; Nakagawa & Kowada, 1983) rectify failures. Consequently, the expected number of failures within the interval $[0, t]$ is $H(t) = \int_0^t h(u)\mathrm{d}u$. When PM actions are performed, the equipment failure process at each interval $[t_i, t_{i+1}]$ is still an NHPP. After the ith PM action; however, the failure intensity becomes $h(t_i) - i\delta \geqslant 0$ for all $i = 1,2,\ldots,n$ as shown in Fig. 1.

According to NHPP, the expected number of failures within the lease period under the proposed PM scheme becomes

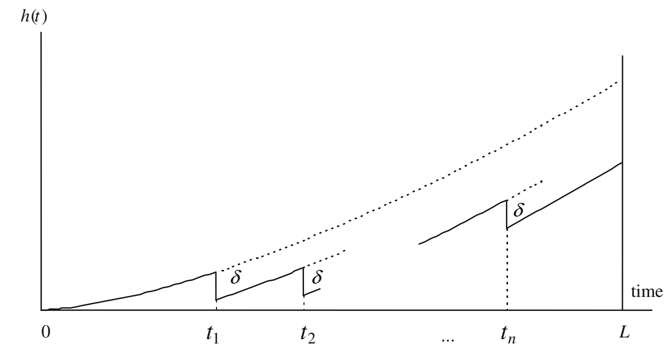

Fig. 1. Preventive maintenance scheme under fixed failure rate reduction.

$$\Lambda \equiv \Lambda(n, \delta, \underline{\mathbf{t}}) = \sum_{i=0}^{n} \int_{t_i}^{t_{i+1}} [h(t) - i\delta]\mathrm{d}t = H(L) - \delta \sum_{i=1}^{n} (L - t_i), \qquad (1)$$

where $\underline{\mathbf{t}} = (t_1, t_2, \ldots, t_n)$ represent the vector of time epochs to perform PM actions. The expected total cost to the lessor within the lease period includes minimal repair cost, penalty cost, and PM cost. As a result, the expected total cost becomes

$$\begin{aligned} C(n, \delta, \underline{\mathbf{t}}) &= [C_m + C_n + C_\tau G(\overline{\tau})]\Lambda + nC_{pm}(\delta) \\ &= KH(L) + nC_{pm}(\delta) - K\delta \sum_{i=1}^{n} (L - t_i), \end{aligned} \qquad (2)$$

where $K = C_m + C_n + C_\tau \int_\tau^\infty \overline{G}(t)\mathrm{d}t$ represents the expected cost for each failure. Without PM actions ($n = 0$), the expected total cost reduces to

$$C_0 \equiv C(0, 0, \underline{\mathbf{t}}; L) = KH(L). \qquad (3)$$

The objective of this study is to find an optimal PM policy $(n^*, \delta^*, \underline{\mathbf{t}}^*)$ for the lessor such that the expected total cost in Eq. (2) is minimized. Note that there are $n + 2$ decision variables (including the number of PM actions n, PM degree δ, and time epochs t_i) in the objective function Eq. (2). In the next section, the properties of the optimal PM policy are investigated and an efficient algorithm is developed based on these properties.

3. The optimal PM policy

Observing Eq. (2), it is clear that there is a trade-off between $nC_{pm}(\delta)$ and $K\delta\sum_{i=1}^{n}(L - t_i)$ in finding the optimal policy since $KH(L)$ is a constant. Therefore, if $nC_{pm}(\delta) - K\delta\sum_{i=1}^{n}(L - t_i) \geq 0$ for all $n > 0$, then preventive maintenance is not worthwhile, which implies $n^* = 0$. In this case, the resulting expected cost becomes $C_0 = KH(L)$. On the other hand, when $nC_{pm}(\delta) - K\delta\sum_{i=1}^{n}(L - t_i) < 0$ for all $n > 0$, n^* exists and the optimal policy is derived based on the following mathematical program:

$$\begin{aligned} &\text{Minimize } C(n, \delta, \underline{\mathbf{t}}) = C_0 + nC_{pm}(\delta) - K\delta \sum_{i=1}^{n} (L - t_i) \\ &\text{Subject to } h(t_i) - i\delta \geq 0 \quad \text{for all} \quad i = 1, 2, \ldots, n. \end{aligned} \qquad (4)$$

Since $h(t)$ is a strictly increasing function of t, the inverse function of the failure rate, h^{-1}, is also a strictly increasing function. Given any $n > 0$ and $\delta > 0$, the following theorem shows the relationship between the optimal time epoch t_i^* and the inverse failure rate function h^{-1}. (Note that all the proofs of the Theorems in this paper are given in the Appendix.)

Theorem 1. *Given any $n > 0$ and $\delta > 0$, if $h(t)$ is a strictly increasing function of t, then $t_i^* = h^{-1}(i\delta)$.*