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Yield curve estimation by kernel smoothing methods

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Abstract

We introduce a new method for the estimation of discount functions, yield curves and forward curves from government issued coupon bonds. Our approach is nonparametric and does not assume a particular functional form for the discount function although we do show how to impose various restrictions in the estimation. Our method is based on kernel smoothing and is defined as the minimum of some localized population moment condition. The solution to the sample problem is not explicit and our estimation procedure is iterative, rather like the backfitting method of estimating additive nonparametric models. We establish the asymptotic normality of our methods using the asymptotic representation of our estimator as an infinite series with declining coefficients. The rate of convergence is standard for one dimensional nonparametric regression. We investigate the finite sample performance of our method, in comparison with other well-established methods, in a small simulation experiment.

JEL classification: C14; G12

Keywords: Coupon bonds; Kernel estimation; Hilbert space; Nonparametric regression; Term structure estimation; Yield curve; Zero coupon

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1. Introduction

The term structure of interest rates is a central concept in monetary and financial economics. Prices of fixed income securities like bonds, swaps, and mortgage backed bonds (MBB's) are functions of the yield curve, and pricing of derivatives also depends on the yield curve. The spread between long and short term interest rates carries information about the level of future interest rates, see for example Campbell and Shiller (1991) and Engsted and Tanggaard (1995). The slope of the yield curve has frequently been used in empirical studies as a predictor of future inflation and national incomes, see Frankel and Lown (1994) and Estrella and Mishkin (1998) for example. Therefore, estimation of yield curves has had a long tradition among financial researchers and practitioners. See Campbell et al. (1997) for further discussion.

A fundamental problem is that the yields to maturity on coupon bonds are not directly comparable between bonds with different maturities or coupons. Thus, there is a need for a standardized way of measuring the term structure of interest rates. One such standard is the yield curve of zero-coupon bonds issued by sovereign lenders.

The construction of this yield curve poses several problems for applied research. First, many governments do not issue longer term (i.e., greater than 1–2 years) zero-coupon bonds. Hence the yield curve must be inferred from other instruments. A simple solution can be derived from the law of one price by assuming the absence of arbitrage. Arbitrage in the bond market will cause the price p of any bond (coupon or zero) with payments $b(\tau_j)$ at time τ_j to be equal to the discounted value of the future cash flow $\pi = \sum_{j=1}^{m} b(\tau_j)d(\tau_j)$, where the discount factor is $d(\tau_j)$ at time τ_j. The future income stream, $b(\tau_1),\ldots,b(\tau_m)$, is assumed known and non-random. The second problem is that, in practice, small pricing errors perhaps due to non-synchronous trading, taxation, illiquidity, and bid–ask spreads necessitate adding an error term to π. The error term should be sufficiently small to ensure that they do not represent (gross) violations of the law of one price (no-arbitrage condition).[1]

The statistical problem we address is to estimate the function $d(\cdot)$ from a sample of coupon paying bonds. Note that, based on a continuous time approximation, we have $d(t) = \exp(-ty(t))$, where $y(t)$ is the yield curve, and $y(t) = \exp(-\int_0^t f(s)\,\mathrm{d}s)$, where $f(t)$ is the forward curve, see Anderson

[1] Some recent studies support this, for example Amihud and Mendelsohn (1991). They found that close to maturity notes and T-bills with identical payment streams can be priced differently, although when taking account of the various transactions costs, there were not, on average, arbitrage opportunities.