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Competitive strategy for on-line leasing of depreciable equipment

Yong Zhang [a], Weiguo Zhang [a], Weijun Xu [a,*], Hongyi Li [b]

[a] School of Business Administration, South China University of Technology, Guangzhou, Guangdong 510640, PR China

[b] Business Administration Faculty, The Chinese University of Hong Kong, Shatin, N.T., Hong Kong

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ABSTRACT

This paper studies the problem of on-line leasing of depreciable equipment. We first discuss deterministic strategies for the problem for situations with and without interest rates, respectively. Competitive ratios are used to evaluate the performance of on-line algorithms. The optimal competitive ratio of the deterministic strategy is therefore obtained. As randomization can boost performance, the randomized strategy for on-line leasing of depreciable equipment with oblivious adversary is further proposed. The main difference between the deterministic algorithm and the randomized algorithm is that the introduction of an interest rate decreases the optimal deterministic competitive ratio while it increases that of the latter. By comparison, we find that the introduction of a depreciation factor improves the competitive performance and the model is more practical for on-line leasing of depreciable equipment. Finally, we conclude that the competitive ratio is closely related to the transaction costs and the profitability in the leasing business.

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1. Introduction

In recent years, it is a hot topic to use competitive analysis and on-line algorithms to study financial optimization problems [1–3]. Competitive analysis compares the performance of on-line algorithms to that of the optimal off-line algorithm and uses a competitive ratio to evaluate the performance of on-line algorithms. Competitive analysis [4,5] has been gaining recognition for being a complementary approach in the analysis of algorithmic decision making under uncertainty. In this approach one assumes that the events are generated by an adversary and the performance of an on-line strategy in which the decision maker has no knowledge about the future events is compared with a corresponding strategy with full knowledge about the entire future events and operates optimally. The problems that the on-line algorithms intend to solve usually have typical dynamic characteristics, which are called on-line problems. Thus, the strategies solving on-line problems are called on-line strategies.

1.1. Competitive analysis

On-line algorithms have been used in many fields such as computer science, economics, operation research, etc. On-line problems had already been investigated in the 1970s and early 1980s but an extensive and systematic study started only when Sleator and Tarjan [4] suggested to compare an on-line algorithm to the optimal off-line algorithm and Karlin et al. [6] coined the term competitive analysis. In general, using competitive ratios for the evaluation of on-line algorithms is called competitive analysis which does not consider the absolute behavior of the algorithm but its relative performance

* Corresponding author.
E-mail addresses: zhangyong8201@gmail.com (Y. Zhang), wgzhang@scut.edu.cn (W. Zhang), wjlk.xu@gmail.com (W. Xu), hongyi@baf.msmail.cuhk.edu.hk (H. Li).

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to the optimal behavior on the same total input sequence. An on-line algorithm receives the input sequence incrementally, one piece at a time. In response to each input the algorithm must generate a corresponding output, without knowing the future inputs. Consider a cost minimization problem P consisting of a set I of inputs, and denote by $Cost_{ALG}$ and $Cost_{OPT}$ the cost of the on-line algorithm ALG and that of the optimal off-line algorithm OPT, respectively. The algorithm ALG is called R-competitive if there exists a constant α such that

$$Cost_{ALG}(I) \leq R \cdot Cost_{OPT}(I) + \alpha, \tag{1.1}$$

for all input sequences I. The smallest R satisfying (1.1) is called the ALG's competitive ratio. Here, the ALG is a deterministic algorithm. Manasse et al. [7] mainly dealt with deterministic on-line algorithms, in which case the definition of being R-competitive is a rather formulated matter. However, Fiat et al. [8] soon realized that randomization could possibly offer the on-line player more power since the moves of the on-line player are no longer certain. In the case of randomized on-line algorithms, costs are taken to be the expected values of the associated random variables [9] and the definition of competitiveness becomes a more subtle issue, depending primarily on the nature of the adversary. In this paper, we are specially interested in an oblivious adversary who must construct the request sequence in advance, but pay for it optimally [9]. Fiat et al. [8] provided a dramatic example of the advantage provided by randomization against this adversary. A randomized algorithm ALG is called R-competitive if there exists a constant α such that

$$E[Cost_{ALG}(I)] \leq R \cdot E[Cost_{OPT}(I)] + \alpha, \tag{1.2}$$

for all input sequences I. When the constant α in (1.1) and (1.2) is less than or equal to zero, we may emphasise that the ALG is strictly R-competitive, a case which we are specifically interested in. Thus, an R-competitive algorithm is guaranteed to incur a cost no larger than R times the smallest possible cost in off-line action for the same input sequence.

As can be seen in the definitions above, the competitive ratio is a measurement of the worst case performance and it is easily understood that the on-line problem is a two-person game between an on-line player and an adversary. In this game the on-line player chooses an on-line algorithm which is known to the adversary, and then the adversary chooses an input sequence. The payoff to the adversary is measured by the performance ratio, i.e., the optimal off-line costs to the on-line costs. Particularly, competitive analysis is very attractive with regard to financial transactions. One major advantage of this approach over the traditional average-case measure is that the need to construct a probabilistic assumption is avoided. This advantage is clearly proved in cases where one is unable to specify precisely the relevant underlying distribution of a stochastic model where the distributional approach might be of little use. Another important feature of the competitive ratio is that it is a relative performance measure. In many situations, the economic and financial agents would prefer to compare their own performance to their peers, rather than to maximize their utility in some absolute sense [5].

1.2. Related research

The method of competitive analysis has been used in many aspects of the financial optimization problems. A large number of theoretical results have been obtained in the literature. Particularly, on-line leasing has been considered widely [10,5,11–13], including the portfolio selection [14–18], the search and one-way trading [19,20], the equipment replacement [21,22], and the stock speculation and gambling [23,24], etc. For the on-line leasing problem, the well-known "ski-rental" example was put forward by Karp [10] in the field of theoretical computer science. We briefly review the basic leasing model and its main conclusion as follows. In a rental activity, let the total periods of actual leases be n, and the costs of renting in each period and the purchase price of the equipment be 1 and a positive integer s, respectively. For the off-line problem, if $s \leq n$, then buy; otherwise rent. For the on-line problem, we consider the following deterministic on-line strategy: rent up to $t-1$ periods and then buy. Denote this on-line strategy by $A(t)$ $(t = 0, 1, 2, \ldots)$, then the optimal on-line strategy is $A(s)$, and its competitive ratio is $2 - 1/s$. A series of researches has been carried out based on this basic model. Karlin et al. [25] made a significant contribution to the on-line analysis for the so-called "the ski-rental problem". They gave a randomized on-line algorithm with a competitive ratio of 1.582 which is optimal. EI-Yaniv et al. [5] pointed out that the investor is often confronted with an important factor, the interest rate i, which may be an essential feature of any reasonable financial model. They analyzed the leasing problem with the interest rate, and showed that the competitive ratio of optimal deterministic algorithm is $1 + (1+i)(1-1/s)(1-si/(1+i))$. Note that if $i = 0$, then the ratio reduces to $2 - 1/s$. Further, they showed that the competitive ratio of the optimal randomized algorithm is $2 - \frac{(s/(s-1))^{\gamma}-2}{(s/(s-1))^{\gamma}-1}$, where $\gamma = \frac{\ln(1-s(1-1/(1+i)))}{\ln(1/(1+i))}$. When $i \to 0$, $\gamma \to s$, and $s \to \infty$, then we have $2 - (e-2)/(e-1) \approx 1.582$. al-Binali [26] built a famous risk-reward framework to analyze the rental problem and the one-way trading problem. The framework allows investors to manage their risks based on their forecasts and risk tolerance. The previous research always avoided possibility distribution assumptions. Fujiwara and Iwama [27] first broke through the customary rule to integrate a possibility distribution assumption into the pure competitive analysis in order to study the on-line leasing problems which include the one-way trading problem [20] as well. Other than the above mentioned methods of competitive analysis, there are many other studies regarding the rental problem. Actually it is an optimal stopping problem. Thus the sequential decision-making method that plays an important role in mathematical statistics can also be used to solve the leasing problem. It is beyond the scope of this paper to discuss these related researches about the on-line leasing problem. Interested readers may find the related references in [28,29].

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