Available online at www.sciencedirect.com
SCIENCE DIRECT®
Physica A 354 (2005) 450–462
PHYSICA A
www.elsevier.com/locate/physa

A risk hedging strategy under the nonparallel-shift yield curve ☆

Pu Gong*, Xubiao He

College of Management, Huazhong University of Science and Technology, 1037 Luoyu Road, Wuhan, Hubei 430074, PR China

Received 30 August 2004; received in revised form 10 January 2005
Available online 23 February 2005

Abstract

Under the assumption of the movement of rigid, a nonparallel-shift model in the term structure of interest rates is developed by introducing Fisher & Weil duration which is a well-known concept in the area of interest risk management. This paper has studied the hedge and replication for portfolio immunization to minimize the risk exposure. Throughout the experiment of numerical simulation, the risk exposures of the portfolio under the different risk hedging strategies are quantitatively evaluated by the method of value at risk (VaR) order statistics (OS) estimation. The results show that the risk hedging strategy proposed in this paper is very effective for the interest risk management of the default-free bond.

PACS: 89.65.Gh; 02.50.−r

Keywords: Nonparallel shift-curve; Risk hedging strategy; Duration; Value at risk

1. Introduction

Duration and convexity are well-known concepts in gap management and asset-liability management in banks and other financial institutions. However, the

☆This project is supported by National Natural Science Foundation of China (70271028).
*Corresponding author. Tel.: +86 27 8755 6489; fax: +86 27 8754 2354.
E-mail address: gongpu@mail.hust.edu.cn (P. Gong).

0378-4371/$ - see front matter
doi:10.1016/j.physa.2005.01.031

simplicity that characterizes both concepts is based on an extreme simplification of interest rate movements, which are defined as parallel and instantaneous shifts in the term structure of interest rates. This unreliable assumption restricts the application of duration and convexity (see Ref. [1]). So, it is of great significance to design a risk hedging strategy under nonparallel shift of the yield curve.

Recent years have seen many refinements in the duration models towards empirical multiple factor duration models which involve several duration measures that look for reality-based relations between interest rate changes. Such an empirical multiple factor duration models could be categorized into directional duration models, partial duration models and polynomial duration models.

The first group, directional duration models, attempts to use movements in the term structure of interest rates to deduce the state variables, observable or not, which govern changes of the whole curve. Cooper [2] introduced a factor to capture the movement of the slope of yield curve, proposed a model that could reflect the variation of the term structure of interest rates. Litterman and Scheinkman [3] showed these three factors of level, steepness, and curvature as basic variables to control the variation of the yield curve, and presented a three-factor model that could describe the movement of term structure of interest rates. These models partially relaxed the parallel-shift assumption of the yield curve, and have broadened the application of duration and convexity. However, for evaluating interest rate risk exposure and hedging against this kind of risk, there are still some problems, the main ones being are the complexity of computation and poor accuracy of the result of computation (see Ref. [4]).

Partial duration models explain interest rate changes by shifts in the level of the different segments into which the term structure is subdivided or by shifts in a limited number of interest rates. Ho [5] proposed the concept of key-rate durations to obtain a measure of the risk exposure of a single bond or a bond portfolio. The key-rate durations represent the price sensitivity to a change of each key rate. Further, Reitano [6,7], and Johnson and Meyer [8] assumed that interest rate changes by parallel shifts in the different segments, then transformed the problem of risk immunization under nonparallel shift of the yield curve into the one under parallel shift of the yield curve. The advantage of this segment methodology is obvious: the idea is simple, and the calculation process is realizable and practicable. But the most difficult problem in practice remains unresolved: how to choose the division of the term structure and the way to match different segments.

Finally, polynomial duration models assume a polynomial fitting of term structure shifts. Nawalkha and Chambers [9] derive a polynomial bond return generating function without requiring that term structure shifts be expressed as a polynomial. Soto [10] built a polynomial duration model and did research on the Spanish government debt market.

Other refined methods have been developed to handle nonparallel shifts of term structure of interest rates, such as value at risk (VaR). VaR is a modern way of financial risk management, which allows various immeasurable subjective factors to be transformed into probability of mathematical statistics. Such feature makes VaR have a far-reaching impact. Although VaR is an inadequate measure within the