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On a two-unit cold standby system considering hardware, human error failures and preventive maintenance

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This paper deals with the study of the stochastic analysis of a two-unit cold standby system considering hardware failure, human error failure and preventive maintenance (PM). All time distributions are considered to be arbitrary. Various measures of reliability of the system are obtained using the regenerative point technique. All these measures are also derived based on time t_0. (Practically PM can be performed after time t_0 from the beginning of the system operation.) Finally a simulation study is carried out to illustrate the result.

1. Introduction

A lot of papers [1–4] have studied the stochastic behaviour of two-unit standby systems, but they did not take into their account the human error failures in spite of the 20%–30% of failures due to human error (see Meister [5]). Many papers [6–15,5] have studied the probabilistic analysis of some systems taking into account the human error failure. In these papers no attention was paid towards the effect of preventive maintenance. The aim of this paper is to study a two-unit cold standby system subject to hardware, human error failures and preventive maintenance. Explicit expressions for the following reliability measures of the system are obtained:

1. The mean time to system failure (MTSF).
2. Pointwise and steady state availability.
3. Busy periods with repair due to hardware failures or human error failure or PM.
4. The profit gain of the system.

Finally a numerical example is presented to illustrate the theoretical results.

2. Assumptions

1. The system consists of two identical units.
2. Initially one unit is operating and the other is in standby case (cold standby).
3. The online unit suffers two types of failures, namely, hardware and human error failure.
4. The switch is perfect and instantaneous.
5. When the online unit goes to time t without failure, it goes under the preventive maintenance (PM) provided that the standby unit is available. If the standby unit is not available, PM is missed until the next time of it.
6. The distributions of all times are arbitrary, except that the distributions of hardware and human error failures have increasing failure rates.
7. After the repair or PM, the unit is as good as new.
8. There is one serverman.

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3. Notations

$A(t)$:	Cdf of time until PM.
$B(t)$:	Cdf of time to accomplish PM.
E_0:	State of the system at $t = 0$.
E:	Set of regenerative states.
$\bar{E}$:	Set of non-regenerative states.
$F_l(t)$ $(l = 1, 2)$:	Cdf of hardware and human error failures respectively.
$G_l(t)$ $(l = 1, 2)$:	Cdf of repair times.
$q_{ij}(t), Q_{ij}(t)$:	Pdf and Cdf of time for the system transits from regenerative state S_i to S_j.
$q_{ij}^{(k)}(t), Q_{ij}^{(k)}(t)$:	Pdf and Cdf of time for the system transits from regenerative state S_i to S_j via the non-regenerative state $S_k \in E$.
$\pi_i(t)$:	Cdf of time to system failure starting from state $S_i \in E$.
m_{ij}:	Contribution to mean sojourn time in state S_i, when system transits direct to S_j.
μ_i:	$\int p$[system sojourns in state S_i for at least time t] dt.
$M_i(t)$:	p [system is up initially in state $S_i \in E$ is up at time t without passing through any other regenerative state or returning to itself through one or more states $\in E$].
$AV_i(t)$:	P [The system is up at time $t \mid E_0 = S_i \in E$].
$B_i^l(t)$:	P [serverman is busy with repair of type $l = 1, 2$ at time t starting from state $S_i \in E$].
$G(t)$:	The expected profit incurred in $(0, t]$.
u:	Dummy variable in Laplace transform (LT).
t_0:	The time at which the preventive maintenance (PM)starts.
$*$:	Symbol for LT.
©:	Symbol for convolution of $f(t)$ and $g(t)$ is $f(t)©g(t) = \int_0^t f(x)\mathrm{d}G(t-x)$.

Symbols for the states of the system

$O/OC_1/OC_2/OC_3/S$:	Unit in normal mode/in normal mode continued from state S_1/in normal mode continued from state S_2/unit in normal mode continued from state S_3/in standby mode.
r_1/r_2:	Unit is in failure mode due to hardware failure and under repair of type 1/unit is in failure mode due to human error failure and under repair of type 2.
wr_1/wr_2:	Waiting for repair of type 1/ waiting for repair of type 2.
R_1/R_2:	Unit is under repair of type 1 continued from earlier state/ unit is under repair of type 2 continued from earlier state.
pm/pmc:	Unit under PM/ unit is under PM continued from earlier state.

Considering these symbols, the system can be in any one of the following states.

$$S_0 = (O, S) \quad S_{c_1} = (OC_1, S) \quad S_{c_2} = (OC_2, S) \quad S_{C_3} = (OC_3, S)$$
$$S_1 = (O, r_1) \quad S_2 = (O, r_2) \quad S_3 = (O, pm) \quad S_4 = (wr_1, R_1)$$
$$S_5 = (wr_2, R_1) \quad S_6 = (wr_1, R_2) \quad S_7 = (wr_2, R_2) \quad S_8 = (wr_1, pmc)$$
$$S_9 = (wr_2, pmc).$$

Up states: $S_0, S_{c_1}, S_{c_2}, S_{c_3}, S_1, S_2$ and S_3. Down States: S_4, S_5, S_6, S_7, S_8 and S_9. One can see that S_0, S_1, S_2 and S_3 are regenerative states, but $S_{c_1}, S_{c_2}, S_{c_3}, S_4, S_5, S_6, S_7, S_8$ and S_9 are non-regenerative states.

4. Transition probabilities and sojourn times

It can be observed that the epoch of entry into any of the states $S_i \in E$ are regenerative point. Let $T_0(\equiv 0), T_1, T_2, \ldots$ denote the epochs at which the system enters any state $S_i \in E$ let X_n denote the state visited at epoch T_n+, i.e just after transition at T_n. Then $\{X_n, T_n\}$ is a Markov-renewal process with state space E and

$$Q_{ij}(t) = P[X_{n+1} = j, T_{n+1} - T_n \leq t \mid X_n = i],$$

is the semi-Markov kernel over E.

The transition probability matrix of embedded Markov-chain is

$$P = (p_{ij}) = (Q_{ij}(\infty) = Q(\infty)),$$

with non-zero elements.

By probabilistic arguments, the non-zero elements P_{ij} are,

$$p_{01} = \int \bar{A}(t)\bar{F}_2(t)\mathrm{d}F_1(t),$$