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A two-stage preventive maintenance policy for a multi-state deterioration system

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ABSTRACT

This paper is to propose a two-stage preventive maintenance (PM) policy for the multi-state deterioration system under periodic inspection and with multiple candidate actions for PM. Such policy is mainly to schedule PM optimally and also on how to choose the action optimally for each PM. The scheduling includes two tasks: to determine after completing each PM when to make the decision and then at it to schedule the next PM optimally. We assume that: (1) such actions except *replacement* are imperfect, (2) the inspection and action times can be ignored, (3) the system can be modeled by a multi-state discrete time Markov chain whose transition probabilities will change and be updated only at the instant after completing each PM, and (4) the risks of such imperfect actions will be updated only at the instant after completing each PM. Finally, an example through simulation is presented to illustrate how such policy can be carried out.

1. Introduction

Many studies deal with how to establish the optimal maintenance policy for the multi-state Markov deterioration system. Such policies can be classified into two classes as follows:

(1) The 1st class deals with such a Markov system whose transition probabilities are time-independent. The maintenance policies will be thus time-independent and state-dependent. They can be further classified into *replacement* policies, the maintenance policies with *replacement* and *repair*, and the maintenance policies with multiple actions. Lam and Yeh [1] proposed an optimal control-limit replacement policy for such a system under continuous inspection to determine the optimal threshold state j^* so that the replacement will be taken upon either j^* or the failure state L whichever occurs first. Lam and Yeh [2] also proposed another optimal control-limit replacement policy under periodic inspection to determine optimally both the inspection interval d^* and the threshold state j^* so that the replacement will be taken whenever the system state X after inspection at nd^* satisfies $j^* \leq X \leq L$. Chiang and Yuan [3,4] proposed also control-limit maintenance policies with *replacement* and *repair* for such a system under continuous and periodic inspections, respectively, to determine two threshold states i^*, j^* (where $1 < i^* < j^* < L$) optimally so that *repair* and *replacement* are taken whenever the system state X after inspection satisfies $i^* \leq X < j^*$ and $j^* \leq X \leq L$, respectively. Chen et al. [5] proposed a maintenance policy with multiple actions also for such a system under inspection at each nd for a fixed d. Such policy is mainly to determine at each nd the maintenance action which has the lowest expected total operating cost during the planning interval.

(2) The 2nd class deals with such a Markov system whose transition probabilities are time-dependent. The maintenance policies in this class will be both time and state dependent. Hontelez et al. [6] proposed a condition-based maintenance policy for such a system whose state at each inspection time can be determined in terms of a so-called deterioration function. Such policy is to determine a control-limit rule $\Pi = [\pi; \sigma_1, \sigma_2, \ldots, \sigma_{\pi-1}]$ with the minimum long-term average cost so that: (1) *repair* is taken if the current state surpasses π and (2) *inspection* will be taken after σ_i periods later if otherwise. Chen et al. [7] proposed a PM policy for such a system under scheduled inspection, whose transition probabilities are determined with the help of a defined aging factor. Such policy is to determine the optimal maintenance period K^* at nd with the lowest average cost rate, so that: (1) PM will be taken at $(n+K^*)d$ if $T(n) > K^*d$, (2) *repair* is taken at $nd+T(n)$ if $T(n) \leq K^*d$, where $T(n)$ denotes the time to failure from nd.

The purpose of this article is to propose a two-stage PM policy with multiple actions for the multi-state Markov deterioration system under periodic inspection. The maintenance actions

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Nomenclature

d	the pre-set inspection period
$n_r d$	the time at which to determine when to take the rth optimal PM
$(n_r+m_r)d$	the time at which the rth optimal PM action is taken
$a_{j,k}$	the candidate action which tries to restore the system from state j to k for $k\in\{1, 2, \ldots, j-1\}$. The replacement is denoted by $a_{L,1}$
$\gamma_l^r(a^{j,k})$	the probability that the system will result to state l after performing $a_{j,k}$ for the rth PM. Such probabilities are named as the risks of $a_{j,k}$
$A(j)$	the set of candidate actions upon state j. That is, $A(1)=\{a_{1,1}\}$,$A(L)=\{a_{L,1}\}$and $A(j)=\{a_{j,k} \mid k=1, \ldots, j-1\}$ for $j=2-,\ldots, L-1$
$C(a_{j,k})$	the cost to take $a_{j,k.}$
R_L	the cost to replace the system
C_I	the cost to make each inspection
$o_n(i)$	the total cost spent during $(nd, (n+1)d)$ due to minimal repairs and system operation given that the system state at nd is i
$\{X_n\}_{n=0,1,\ldots}$	a Markov chain where X_n denotes the system state identified at nd

except *replacement* are imperfect. Their risks are taken into consideration. Both the transition probabilities and the risks will change and be updated only at the instant after completing each PM.

2. Policy assumptions

The deterioration system considered herein is inspected at each nd for $n=1, 2, 3, \ldots$. It can be classified into L states in the order $1(\text{new})<2<\ldots<L(\text{complete failure})$ to reflect their relative degree of deterioration. It is modeled into an L-state Markov chain $\{X_n\}_{n\geq 0}$ where X_n denotes the system state at nd. For our purpose, we may first assume the transition probabilities $p_{j,h}^n$ of the Markov chain are time-independent in case that the system is under no maintenance at all. Fig. 1 illustrates the state transitions of the system under no maintenance. Whenever the maintenance are taken into consideration, we may further assume that such transition probabilities change at the instant after completing each PM and keep the same until the completion of the next PM. We thus denote the transition probabilities after completing the $(r-1)$th action by $p_{j,h}^{(r)}$ for $r=1, 2, \ldots$. The transition matrix $P^{(r)}=[p_{j,h}^{(r)}]$ is in upper-triangular form to reflect the system deterioration. That is, such $p_{j,h}^{(r)}$ satisfies $p_{j,h}^{(r)}=0$ for each pair $j>h$. To carry out the PM policy for such a system, the following assumptions are further required:

A1: The actions *do-nothing* and *replacement* are taken upon $X_n=1$ and $X_n=L$, respectively.
A2: For the action $a_{j,k}$ which is chosen as the rth PM, the system will be restored to the desired state k with probability $\gamma_k^r(a_{j,k})$ and to the undesired state l with probability $\gamma_l^r(a_{j,k})$ for $l\neq k$, s.t $\sum_{l\geq k}\gamma_l^r(a_{j,k})=1$.
A3: The times spent on both inspection and maintenance can be ignored.
A4: State F which denotes the system's sudden and temporary interruption due to fatal shock may also occur. Such F announces itself and the probability that F occurs at nd can be ignored for each $n=1, 2, 3, \ldots$. That is, $p\{X_n=F\}=0\forall n$.

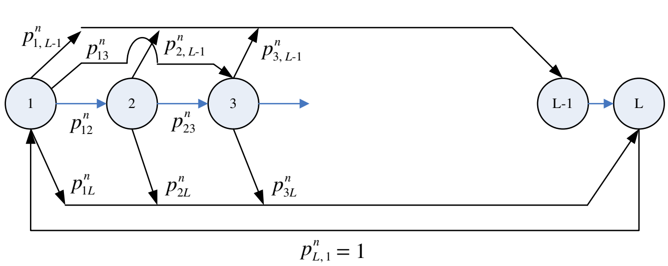

Fig. 1. The state transitions of the system under no maintenance at nd.

Whenever F occurs, the minimal repair is taken immediately in negligible time and costless.

3. The PM policy for the deterioration system

3.1. The PM planning

The proposed PM policy is mainly on how to both schedule the PM optimally and choose the action optimally for each coming PM. The scheduling is carried out in two phases. Phase 1 is to determine either at starting time 0 or after completing the $(r-1)$th PM when to make the decision. Phase 2 is to determine at the decision time when to perform the rth PM optimally. Suppose that $n_s d$ and $(n_s+m_s)d$ are the observed sth decision and sth PM completion times respectively for each $s=1, 2, \ldots, r-1$. The rth PM will be planned at $(n_{r-1}+m_{r-1})d$ in the following two stages:

Stage I: Determine at $(n_{r-1}+m_{r-1})d$ when to make the decision for the rth PM: the rule is to set the threshold state for decision ω_r in advance. In case $r=1$ and so $(n_{r-1}+m_{r-1})d=0$, the decision will be made whenever the system is first found surpassing ω_1 by inspection. In case $r\geq 2$, the decision will be made whenever the system is first found surpassing ω_r since $(n_{r-1}+m_{r-1})d$ by inspection.

Stage II: Determine at $n_r d$ on how to choose the action for the rth PM optimally and schedule the rth PM at it optimally where $n_r d$ be the observed time at which the rth decision should be made:

Stage II-1: Suppose that the rth PM will be taken at nd while $X_n=j$ and $a_{j,k}$ is chosen. We let $\phi_r(a_{j,k})$ denote the expected total cost over $[nd, (n+1)d]$ by

$$\phi_r(a_{j,k})=C(a_{j,k})+\sum_{l=k}^{j}\gamma_l^r(a_{j,k})o_n(l) \tag{1}$$

We thus need to estimate $\gamma_l^r(a_{j,k})$ at $n_r d$ too. The optimal action a_j^* then will be chosen according to

$$a_j^*=\arg\underset{\forall a_{j,k}\in A(j)}{Min}\phi_r(a_{j,k}) \tag{2}$$

Stage II-2: Determine at $n_r d$ while $X_{n_r}=x_{n_r}$ when to perform the rth PM optimally: suppose that we choose $j\in\{x_{n_r},\ldots,L-1\}$ as the threshold state for the rth PM. That is, the rth PM will be taken at $(n_r+M_j)d$ where M_j is the first integer that $X_{n_r+M_j}\in\{j,j+1,\ldots,L-1\}$. Suppose that $a^*_{x_{n_r+M_j}}$ is the chosen action for the rth PM according to Stage I and let the associated expected cost rate $\tau_{n_r,x_{n_r}}(j)$ over $[0,(n_r+M_j+1)d]$ by

$$\tau_{n_r,x_{n_r}}(j)=\frac{E(C(n_r+M_j,a^*_{x_{n_r+M_j}}))}{E(M_j+1)}$$