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Preventive-maintenance policy for leased products under various maintenance costs

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ABSTRACT

This paper investigates the effects of preventive-maintenance cost functions on the optimal preventive-maintenance policy for a leased product with Weibull life-time distribution. During the lease period, any product failures are rectified by minimal repairs and may incur a penalty to the lessor, if the time duration for performing a minimal repair exceeds a pre-specified time limit. To reduce repair costs and possible penalty, preventive-maintenance actions are scheduled in the lease contract. The objective of this paper is to derive the optimal preventive-maintenance schedule and maintenance degrees such that the expected total cost is minimized. Some structural properties of the optimal policy are obtained and an efficient algorithm is provided to search for the optimal policy. For the cases where the preventive maintenance cost is a constant or a linearly increasing function, the effects of the preventive-maintenance cost function on the optimal policy are investigated in detail both theoretically and numerically.

1. Introduction

This paper derives the optimal preventive-maintenance policy for a leased product with Weibull life-time distribution and investigates the effects of preventive-maintenance cost function on the optimal policy. Due to the increase in complexity of products/systems and rapid advances in technological innovation, performing maintenance actions for such complex products now requires expensive equipments and special professional technicians, which is not economical for a company to keep. Therefore, there is a trend to lease products instead of owning them (Desai & Purohit, 1998; Fang & Huang, 2008). For a leased product, the maintenance actions are usually provided by the lessor (the one who owns the product) and specified in a lease contract to ensure that the product could fulfill its intended performance requested by the lessee (the one leasing the product).

In general, there are two types of maintenance considered in a lease contract – corrective maintenance (CM) and preventive-maintenance (PM). CM actions are employed to rectify failed products back to operational status, and PM actions are used to improve the operational status of a product to avoid failures (Barlow & Hunter, 1960; Valdez-Flores & Feldman, 1989). The articles in Dekker (1996), Dekker and Scarf (1998), Pieskalla and Voelker (1976), Sherif and Smith (1981) and Valdez-Flores and Feldman (1989) are excellent reviews of maintenance models for products subject to stochastic failures. In developing a maintenance model, minimal repair is the most commonly used corrective maintenance action to restore a failed product (Nakagawa, 1981; Nakagawa & Kowada, 1983), since the failure rate of the product remains unchanged after performing a minimal repair. Various maintenance models involving minimal repair can be found in Boland and Proschan (1982), Nakagawa (1981), Nakagawa and Kowada (1983) and Sheu (1991).

For a leased product, minimal repairs are carried out by the lessor to restore a failed product back to its operational condition. When the time required to perform a minimal repair exceeds a pre-specified amount of time, there is a penalty to the lessor to compensate the loss to the lessee. Therefore, there is a need for the lessor to provide some remedial measures to avoid the costs of minimal repairs and penalty incurred by product failures.

To reduce the number of product failures and possible penalties within the lease period, PM actions are widely employed since the cost for carrying out a planned PM action is usually less than the cost incurred by a product failure. Various PM models have been proposed for different situations such as perfect or imperfect PM (Barlow & Hunter, 1960; Jack & Dagpunar, 1994; Pham & Wang, 1996), and periodical or sequential PM (Chun, 1992; Jack & Dagpunar, 1994; Yeh & Lo, 2001). For the imperfect PM, the maintenance degree for each PM action is characterized by age-reduction or failure rate-reduction methods (Barlow & Hunter, 1960; Pham & Wang, 1996). Using the age-reduction method, the age of the product after taking a PM action becomes a certain amount of time younger than before. On the other hand, using the failure rate-reduction method, the failure rate of the product is reduced by a certain amount after a PM action.

In this paper, the age-reduction method is adopted since it is easily measured and implemented in practice. Using the

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age-reduction method, a mathematical cost model is developed and the optimal PM policy is derived such that the expected total cost in the lease period is minimized. Furthermore, the effects of the PM cost function on the optimal PM policy are investigated in detail.

The remainder of this paper is organized as follows. The mathematical model is developed in Section 2 for the case when the failure density is Weibull. In Section 3, the properties of the optimal PM policy are investigated, and an efficient algorithm is provided for searching the optimal policy when PM cost is a general function of the maintenance degree. Furthermore, closed-form solutions of the optimal policy are obtained for the special case when PM cost function is constant or linearly increasing. The performance of the optimal PM policy is evaluated through numerical examples in Section 4. Finally, some conclusions are drawn in the last section.

2. Mathematical formulation

Consider that a new product with Weibull life-time distribution is leased for a period L. Within the lease period, any failure of the product is minimally repaired by the lessor. After minimal repair, the product is operational but the failure rate of the product remains the same as that just before failure. It is well-known that the failure rate function of a Weibull distribution with scale parameter $\lambda > 0$ and shape parameter $\beta > 0$ is given by $h(t) = \lambda\beta(\lambda t)^{\beta-1}$. Hence, when minimal repairs are carried out upon product failures, the failure rate function $h(t)$ of the product continuously decreases in t if $\beta < 1$, increases if $\beta > 1$, and is a constant if $\beta = 1$. If a product fails during the lease period, minimal repair is performed immediately with a fixed repair cost $C_m > 0$ and requires a random amount of repair time t_m, which follows a general cumulative distribution function $G(t_m)$. If the repair time exceeds a predetermined value τ, then there is a penalty $C_\tau > 0$ to the lessor. That is, a possible penalty of C_τ to the lessor may occur with probability $\overline{G}(\tau) = 1 - G(\tau)$ at each failure, when the repair time is longer than τ.

In addition to minimal repairs at failures, there are n PM actions to be specified in the lease contract. That is, the lessor will perform n PM actions sequentially at time epochs T_i (where $0 \leqslant T_1 \leqslant T_2 \leqslant \cdots \leqslant T_n \leqslant L$) with maintenance degrees $x_i > 0$, where $\sum_{j=1}^{i} x_j \leqslant T_i$ for $i = 1,2,\ldots,n$. After performing the ith PM action at time T_i with degree x_i, the age of the product is x_i units of time younger than before. The purpose of performing PM actions is to reduce the total number of product failures and the resulting costs of minimal repairs and penalty within the lease period. However, performing a PM action also incurs a certain amount of cost. Let $C_p(x)$ be the cost for carrying out a PM action with maintenance degree x. In general, the cost of a PM action is a non-negative and non-decreasing function of maintenance degree x; that is, $C_p'(x) \geqslant 0$.

Under minimal repairs, the failure process of the product can be represented by a non-homogeneous Poisson process (NHPP) with failure intensity $h(t)$ when PM actions are not performed (Pham & Wang, 1996; Pieskalla & Voelker, 1976). Without PM actions, the expected number of failures of the product within the interval $[0,t]$ becomes $\int_0^t h(t)dt = (\lambda t)^\beta$. When PM actions are taken at time epochs T_i, the failure process of the product in each interval $[T_i,T_{i+1})$ is still an NHPP after the ith PM action, but the failure intensity becomes $h\left(t - \sum_{j=1}^{i} x_j\right)$ for $t \in [T_i,T_{i+1})$. Assuming that the times required for carrying out the minimal repair and PM actions are negligible compared to the leased period L. Then, under the sequential PM scheme, the expected number of failures during the lease period becomes

$$\Lambda(L) = \lambda^\beta \sum_{i=0}^{n} \left\{ \left[T_{i+1} - \sum_{j=1}^{i} x_j \right]^\beta - \left[T_i - \sum_{j=1}^{i} x_j \right]^\beta \right\}, \qquad (1)$$

where $T_0 = 0$ and $T_{n+1} = L$.

As a result, the expected total cost to the lessor under the aforementioned lease contract includes the minimal repair cost, penalty cost, and PM cost. To simplify the notations, let $\mathbf{X} = (x_1,x_2,\ldots,x_n)$ and $\mathbf{T} = (T_1,T_2,\ldots,T_n)$. Then, the expected total cost to the lessor under PM policy $(n,\mathbf{X},\mathbf{T})$ becomes

$$C(n,\mathbf{X},\mathbf{T}) = \sum_{i=1}^{n} C_p(x_i) + \left[C_m + C_\tau \overline{G}(\tau)\right]\Lambda(L) = \sum_{i=1}^{n} C_p(x_i) + \lambda^\beta \left[C_m + C_\tau \overline{G}(\tau)\right] \sum_{i=0}^{n} \left\{ \left[T_{i+1} - \sum_{j=1}^{i} x_j \right]^\beta - \left[T_i - \sum_{j=1}^{i} x_j \right]^\beta \right\} \qquad (2)$$

with $T_0 = 0$ and $T_{n+1} = L$. Our objective here is to find an optimal PM policy $(n^*,\mathbf{X}^*,\mathbf{T}^*)$ for the lessor such that the expected total cost in Eq. (2) is minimized under the constraints of $T_i \geqslant \sum_{j=1}^{i} x_j$ for $i = 1,2,\ldots,n$ and investigate the effects of $C_p(x)$ on the optimal PM policy $(n^*,\mathbf{X}^*,\mathbf{T}^*)$.

3. Optimal PM policy

Obviously, when $\beta \leqslant 1$, the failure rate function of the product is non-increasing in t; that is, $h(t_1) \geqslant h(t_2)$ *for all* $0 \leqslant t_1 \leqslant t_2$. Hence, reducing the age of the product by PM actions is not desired in this case. That is, any policy with $n^* = 0$ is optimal when $\beta \leqslant 1$ and the resulting expected total cost is $C(0,\mathbf{X},\mathbf{T}) = \left[C_m + C_\tau \overline{G}(\tau)\right](\lambda L)^\beta$. On the other hand, PM actions may be required to reduce the age of the product to avoid failures in the lease period when $\beta > 1$. Therefore, in the following discussion, we will focus on this case. Based on the objective function (2), we first derive some structural properties of the optimal PM policy for a lease contract and then investigate the optimal PM policy in detail under various PM cost functions.

To investigate the optimal maintenance schedule $\mathbf{T}^*$ when n and $\mathbf{X}$ are pre-specified, we take the first partial derivative of Eq. (2) with respect to T_i as follows:

$$\frac{\partial C(n,\mathbf{X},\mathbf{T})}{\partial T_i} = \beta\lambda^\beta \left[C_m + C_\tau \overline{G}(\tau)\right] \left\{ \left[T_i - \sum_{j=1}^{i-1} x_j \right]^{\beta-1} - \left[T_i - \sum_{j=1}^{i} x_j \right]^{\beta-1} \right\}. \qquad (3)$$

From Eq. (3), the following theorem holds.

Theorem 1. *Given any* $n > 0$ *and* $\mathbf{X} > \mathbf{0}$*, the optimal maintenance schedule* $\mathbf{T}^*$ *is* $T_i^* = \sum_{j=1}^{i} x_j$ *for i = 1,2,…,n, when* $\beta > 1$*.*

Proof. When $\beta > 1$ and $\mathbf{X} > \mathbf{0}$, from Eq. (3), we have $\frac{\partial C(n,\mathbf{X},\mathbf{T})}{\partial T_i} > 0$ for all $i = 1,2,\ldots,n$, since $\left[T_i - \sum_{j=1}^{i-1} x_j\right]^{\beta-1} > \left[T_i - \sum_{j=1}^{i} x_j\right]^{\beta-1}$. This implies that $C(n,\mathbf{X},\mathbf{T})$ is an increasing function of T_i. Hence, under the constraints $T_i \geqslant \sum_{j=1}^{i} x_j$ for $i = 1,2,\ldots,n$, it is clear that $T_i^* = \sum_{j=1}^{i} x_j$ when $\beta > 1$. □

Theorem 1 shows that PM actions should be scheduled at the time epochs $x_1, x_1 + x_2, x_1 + x_2 + x_3, \ldots, x_1 + x_2 + \cdots + x_n$ when $\beta > 1$. In other words, PM actions are performed to restore the product back to its original condition if the maintenance degrees are pre-specified. Applying this optimal PM schedule, the objective function becomes

$$C(n,\mathbf{X}) = \sum_{i=1}^{n} C_p(x_i) + \lambda^\beta \left[C_m + C_\tau \overline{G}(\tau)\right] \left[\sum_{i=1}^{n} x_i^\beta + \left(L - \sum_{i=1}^{n} x_i \right)^\beta \right]. \qquad (4)$$