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Age-based preventive maintenance for passive components submitted to stress corrosion cracking

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ABSTRACT

This paper deals with an age-based preventive maintenance for critical systems or structures subject to a gradual degradation phenomenon such as stress corrosion cracking. We analyze a system subjected to different cracks. A crack can be only detected when its length exceeds a detection threshold. When the length of the crack reaches a given threshold, the system fails. The length of the crack is modeled using a gamma process. Furthermore, when the number of cracks detected in the system attains a fixed value, the system fails. Corrective maintenance actions are performed after a system failure. A preventive maintenance is performed when the age of the system is T. Maintenance actions replace the system by a new one with an associated cost. The problem is to determine an optimal planned replacement time T, minimizing the expected cost rate of the system. The analytical solution to the problem is obtained under some general assumptions. A numerical example is shown to illustrate the problem.

1. Introduction

A corrosive environment is one of the major sources of degradation for passive components or structures. In many cases, the devices are exposed to a deterioration that increases in time and can cause serious damages in the functioning of the system. Stress corrosion cracking (SCC) is a process that appears from the combined influence of tensile stress and corrosive environment and involves the initiation and propagation of cracks. In industrial installations it has been attributed to many engineering failures and can lead to practical important effects from an economic or safety point of view and for power plants' lifetime estimation [1]. For this reason, the SCC process has been extensively analyzed in the last decades and some models have been developed which address the failure analysis of components subject to SCC and the way to increase the safety in the systems (see e.g. [2–4]). Boursier et al. [2] analyzed Alloy 600 for steam generator tubing in pressurized water reactors and described the different stages of the cracking.

1. The period of initiation of the crack or incubation period during which the crack has not yet appeared. It ends when the environment becomes appropriate for SCC and the crack initiates.
2. The slow-propagation stage hereafter refereed as propagation period. The crack appears and it grows until it reaches a critical size.
3. The rapid-propagation stage which leads to failure or rupture of the component. All this stage is supposed to be a critical stage corresponding to a "failed" condition (rupture) for the considered components.

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As is shown in [5], the prediction of the crack growth (i.e. the prediction of the degradation level) is important in the reliability and durability analysis of critical components in order to derive residual lifetimes or failure times and to schedule inspection and maintenance tasks as repairs or replacements. According to Singpurwalla [6], stochastic models are very effective tools to model time dependent degradation like SCC when the final purpose is to predict the failure time or the residual lifetime. This can be explained by the uncertainty contained in the operating feedback data due to operational conditions or by the scatter observed in data under the same environmental conditions, that are attributed to the inhomogeneous material properties. For SCC, Zhang et al. [7] showed that the crack initiation time can be considered as an exponentially distributed random variable (r.v.) and considered the Weibull distribution for the modeling of the crack velocity.

In this paper, the propagation stage is modeled using a stochastic process which represents the crack size evolution in time. The difference of the crack size between two successive inspection times is considered as a random variable. Because of its modeling capacity and its mathematical tractability, the gamma process is considered. The gamma process is a stochastic process with independent non-negative increments having a gamma distribution with identical scale parameter. The work of [8] surveys the application of gamma processes in maintenance.

A SCC model with exponential r.v. for initiation and gamma stochastic process for propagation has been proposed and applied in a previous work for Electricité de France in [9] where two particular cases are analyzed: first an unique component affected by several cracks and second n components on which only one crack is taken into account (usually the largest crack). In both cases and from the available data-set, the parameter of the exponential distribution (for initiation) and the parameters of the gamma process (for propagation stage) are jointly estimated by maximization of the likelihood function. Scarf et al. [10] described a crack growth model where a complex system is subject to deterioration due to fatigue crack growth. The stochastic model is also fitted to a data set on crack depths. Laycock et al. [11] explained the underlying mathematics behind the most commonly used techniques in the statistical analysis of crack growth data.

The present paper adds a new dimension to the SCC models by introducing a maintenance policy for these models. We analyze a system subject to different cracks. The deterioration in the system is related to the length of the cracks. A crack is detected when its length exceeds a detection threshold. Furthermore, when the length of the crack exceeds a failure threshold, the system fails and it has to be replaced immediately. The length of the crack is modeled using a gamma process. Furthermore, the system cannot work satisfactorily and fails when the number of cracks detected in the system exceeds a limited value. Since it is generally less costly to replace a system before it has failed, preventive maintenance policies are introduced to avoid unplanned replacements of the system. In this work, the preventive maintenance is related to the age of the system. So, if the age of the system exceeds a fixed value T, a preventive maintenance is performed. Corrective maintenance is performed when the system fails, that is, when the length of a crack reaches a certain value or when the number of detected cracks in the system exceeds a limit. After a maintenance action, the system is replaced by a new one. Costs are associated with the maintenance actions and the objective is to determine an optimal planned replacement time T which minimizes the long-run average cost per unit time.

Let $\{X(t), t \geq 0\}$ be a regenerative process, we denote by $S_1, S_2, S_3, \ldots$ the regeneration epochs of the process and by $L_i = S_i - S_{i-1}, i = 1, 2, \ldots$ the length of the i-th renewal cycle. If a cost structure is imposed on the regenerative process $\{X(t), t \geq 0\}$ and denoting by $C(t)$ the cost of the system at time t and by C_i the total cost in the i-th renewal cycle, it is well known that (see [12] pg. 41)

$$\frac{C(t)}{t} \longrightarrow \frac{E[C_1]}{E[L_1]}, \tag{1}$$

with probability one, that is, the long-run average cost per unit time is equal to the expected cost in a cycle divided by the expected length of the cycle for almost any realization of the process. The criterion of long-run average cost is widely used in the reliability literature and we shall use (1) as the objective function to minimize.

This paper is structured as follows. The definition of the problem is specified in Section 2. In Section 3, the problem is formulated. Section 4 is focused on the search of the optimal replacement time T and some particular examples of the model are showed. Section 5 presents some numerical examples of the model and Section 6 concludes.

2. Problem definition

We consider a maintenance model of a system subject to cracks, where corrective/preventive maintenances take place according to the following scheme.

1. The system starts working at time 0. This system is subject to cracks that arrive following a Poisson process with rate λ. We denote by $T_i, i = 1, 2, \ldots$, the arrival time (or the initiation time) of the i-th crack where $T_0 = 0$.
2. We denote by $X_t^i,\ i = 1, 2, \ldots$ the length of the i-th crack t units of time after its initiation. We assume that X_t^i follows a homogeneous gamma process with shape and scale parameters given by αt and β respectively, that is, for $s < t$, the density function of the increment of the length of the crack i is given by

$$f_{\alpha(t-s),\beta}(x) = \frac{1}{\Gamma(\alpha(t-s))}\beta^{\alpha(t-s)}x^{\alpha(s-t)-1}e^{-\beta x}, \quad x \geq 0,$$