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Forecasting the yield curve in a data-rich environment: A no-arbitrage factor-augmented VAR approach[☆]

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ABSTRACT

This paper suggests a term structure model which parsimoniously exploits a broad macroeconomic information set. The model uses the short rate and the common components of a large number of macroeconomic variables as factors. Precisely, the dynamics of the short rate are modeled with a Factor-Augmented Vector Autoregression and the term structure is derived using parameter restrictions implied by no-arbitrage. The model has economic appeal and provides better out-of-sample yield forecasts at intermediate and long horizons than a number of previously suggested approaches. The forecast improvement is highly significant and particularly pronounced for short and medium-term maturities.

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1. Introduction

Traditional models of the term structure decompose yields into a set of latent factors. These models commonly provide a good in-sample fit to the data (e.g. Nelson and Siegel (1987), Knez et al. (1994) and Dai and Singleton (2000)) and can also be used to predict interest rates out-of-sample (e.g. Duffee (2002) and Diebold and Li (2006)). While providing a good statistical fit, however, the economic meaning of such models is limited since they disregard the relationships between macroeconomic variables and interest rates. In this paper, I suggest a model which has both economic appeal and superior predictive ability for yields as compared to traditional approaches.

In a widely recognized paper, Ang and Piazzesi (2003) augment a standard three-factor affine term structure model with two macroeconomic factors that enter the model through a Taylor-rule type of short rate equation. They find that the macro factors account for a large share of the variation in interest rates and also improve yield forecasts. Inspired by this finding, a vivid literature has emerged lately that explores different approaches to jointly model the term structure and the macroeconomy. Examples for such models are Hördahl et al. (2006), Diebold et al. (2006) and Dewachter and Lyrio (2006). While these latter studies consistently find that macroeconomic variables are useful for explaining and/or forecasting government bond yields, they only exploit very small macroeconomic information sets. Yet, by limiting the analysis to only a few variables, other potentially useful macroeconomic information is being neglected.[1]

This is particularly important for term structure modeling as a recent literature argues that the central bank acts in a "data-rich environment" (Bernanke and Boivin, 2003). This means that the monetary policy authority bases its decisions upon a broad set of conditioning information rather than only a few key aggregates. Consistent with this argument, a number of studies have found that factors which by construction summarize the comovement in

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[1] Note that the macroeconomic factors in Ang and Piazzesi (2003) are the principal components extracted from a group of four real and three nominal variables, respectively. Accordingly, these authors employ a somewhat larger macroeconomic information set than the other studies referred to above.

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a large number of macroeconomic time series help to explain and forecast the evolution of short-term interest rates (e.g. Bernanke and Boivin (2003), Giannone et al. (2004) and Favero et al. (2005)). In related work, Bernanke et al. (2005) suggest to combine the advantages of factor modeling and structural VAR analysis by estimating a joint vector-autoregression of the short-term interest rate and factors extracted from a large cross-section of macro time series. They label this approach a "Factor-Augmented VAR" (FAVAR) and use it to analyze the dynamics of the short rate and the effects of monetary policy on a wide range of macroeconomic variables.

In this paper, I take the approach of Bernanke et al. (2005) a step further and employ the FAVAR model to study the dynamics of the entire yield curve within an arbitrage-free model. Precisely, I suggest a model that has the following structure. A Factor-Augmented VAR is used to describe the dynamics of the short-term interest rate conditional on a large macroeconomic information set. Given the dynamics of the short rate, the term structure of interest rates is then derived using parameter-restrictions implied by no-arbitrage. In sum, my model is an affine term structure model that has a Factor-Augmented VAR as the state equation, i.e. the short rate and the common components of a large number of macro time series represent the factors which drive the variation of yields. I label this approach a No-Arbitrage Factor-Augmented VAR.

Estimation of the model is in two steps. First, I extract common factors from a large macroeconomic dataset using the method suggested by Stock and Watson (2002a,b) and estimate the parameters governing their joint dynamics with the monetary policy instrument in a VAR. Second, I estimate a no-arbitrage vector autoregression of yields on the exogenous pricing factors. Specifically, I obtain the price of risk parameters by minimizing the sum of squared fitting errors of the model following the nonlinear least squares approach of Ang et al. (2006). Altogether, estimation of the model is fast and it is thus particularly useful for recursive out-of-sample forecasts.

The results of the paper can be summarized as follows. The No-Arbitrage FAVAR model based on four macro factors and the short rate fits the US yield curve well in-sample. More importantly, the model shows a strikingly good ability to predict yields out-of-sample. In a recursive out-of-sample forecast exercise, the No-Arbitrage FAVAR model is found to provide superior forecasts with respect to a number of benchmark models which have previously been suggested in the literature. Except for extremely short forecast horizons and very long maturities, the model significantly outperforms the random walk, a standard three-factor affine model, the model suggested by Bernanke et al. (2004) which employs individual macroeconomic variables as factors, and the model recently put forth by Diebold and Li (2006) which has been documented to be particularly useful for interest rate predictions. A subsample analysis reveals that the No-Arbitrage Factor-Augmented VAR model performs particularly well in periods when interest rates vary a lot.

The paper is structured as follows. In Section 2, the No-Arbitrage Factor-Augmented VAR model is presented and its parametrization discussed. Section 3 describes the estimation of the model. In Section 4, I document the in-sample fit of the model and then discuss the results of the out-of-sample forecasts in Section 5. Section 6 concludes.

2. The model

Economists typically think of the economy as being affected by monetary policy through the short term interest rate. At the same time, the central bank is often assumed to set the short rate as a function of the overall state of the economy, characterized e.g. by the deviations of inflation and output from their desired levels. Bernanke et al. (2005) point out that theoretical macroeconomic aggregates as output and inflation might not be perfectly observable neither to the policy-maker nor to the econometrician. Instead, they argue that the observed macroeconomic time series should be thought of as noisy measures of economic concepts such as aggregate activity or inflation. Accordingly, these concepts should be treated as unobservable in empirical work so as to avoid confounding measurement error or idiosyncratic dynamics with fundamental economic shocks.

Bernanke et al. (2005) therefore suggest to extract a few common factors from a large number of macroeconomic time series variables and to study the mutual dynamics of monetary policy and the key economic aggregates by estimating a joint VAR of the factors and the policy instrument, an approach which they label "Factor-Augmented VAR" (FAVAR). This approach can be summarized by the following equations:

$$X_t = \Lambda_F F_t + \Lambda_r r_t + e_t \tag{1}$$

$$\begin{pmatrix} F_t \\ r_t \end{pmatrix} = \mu + \Phi(L) \begin{pmatrix} F_{t-1} \\ r_{t-1} \end{pmatrix} + \omega_t. \tag{2}$$

X_t denotes a $M \times 1$ vector of period-t observations of the observed macroeconomic variables, Λ_F and Λ_r are the $M \times k$ and $M \times 1$ matrices of factor loadings, r_t denotes the short-term interest rate, F_t is the $k \times 1$ vector of period-t observations of the common factors, and e_t is an $M \times 1$ vector of idiosyncratic components. Moreover, $\mu = (\mu_f', \mu_r)'$ is a $(k+1) \times 1$ vector of constants, $\Phi(L)$ denotes the $(k+1) \times (k+1)$ matrix of order-p lag polynomials and ω_t is a $(k+1) \times 1$ vector of reduced form shocks with variance covariance matrix Ω. Since affine term structure models are commonly formulated in state-space from, I rewrite the FAVAR in Eq. (2) as

$$Z_t = \boldsymbol{\mu} + \boldsymbol{\Phi} Z_{t-1} + \boldsymbol{\omega}_t, \tag{3}$$

where $Z_t = (F_t', r_t, F_{t-1}', r_{t-1}, \dots, F_{t-p+1}', r_{t-p+1})'$, and where $\boldsymbol{\mu}$, $\boldsymbol{\Phi}$, $\boldsymbol{\omega}$ and $\boldsymbol{\Omega}$ denote the companion form equivalents of μ, Φ, ω, and Ω, respectively. Accordingly, the short rate r_t can be expressed in terms of Z_t as $r_t = \delta' Z_t$ where $\delta' = (\mathbf{0}_{1\times k}, 1, \mathbf{0}_{1\times(k+1)(p-1)})$.

2.1. Adding the term structure

The term structure model which I suggest is built upon the idea that the Federal Reserve bases its decisions on a large set of conditioning information and that the dynamics of the short-term interest rate are therefore well described by a Factor-Augmented VAR. Accordingly, yields are driven by the policy instrument as well as the main shocks hitting the economy which are proxied by the factors that capture the bulk of common variation in a large number of macroeconomic variables. I thus employ the FAVAR in Eq. (3) as the state equation of my term structure model. To make the model consistent with the assumption of no-arbitrage, I further impose restrictions on the parameters governing the impact of the state variables on the yields of different maturity. More precisely, I model the nominal pricing kernel as

$$\begin{aligned} M_{t+1} &= \exp\left(-r_t - \frac{1}{2}\lambda_t'\Omega\lambda_t - \lambda_t'\omega_{t+1}\right), \\ &= \exp\left(-\delta' Z_t - \frac{1}{2}\lambda_t'\Omega\lambda_t - \lambda_t'\omega_{t+1}\right), \end{aligned} \tag{4}$$

where λ_t are the market prices of risk. Following Duffee (2002), these are commonly assumed to be affine in the underlying state variables Z, i.e.

$$\lambda_t = \lambda_0 + \lambda_1 Z_t. \tag{5}$$