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Automatic synthesis of uncertain models for linear circuit simulation: A polynomial chaos theory approach

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ABSTRACT

A generalized and automated process for the evaluation of system uncertainty using computer simulation is presented. Wiener–Askey polynomial chaos and generalized polynomial chaos expansions along with Galerkin projections, are used to project a resistive companion system representation onto a stochastic space. Modifications to the resistive companion modeling method that allow for individual models to be produced independently from one another are presented. The results of the polynomial chaos system simulation are compared to Monte Carlo simulation results from PSPICE and C++. The comparison of the simulation results from the various methods demonstrates that polynomial chaos circuit simulation is accurate and advantageous. The algorithms and processes presented in this paper are the basis for the creation of a computer-aided design (CAD) simulator for linear networks containing uncertain parameters.

1. Introduction

Representation of uncertainty, when applied to circuit simulation, can be a powerful tool for producing and designing robust systems. Various methods have been proposed which can be used to quantify and propagate uncertainty. There are interesting examples of research in which Artificial Intelligence techniques are applied to provide a simplified or qualitative definition of the physics of systems when uncertainty is involved. Reviews of this topic can be found in [1–6]. Similar issues have been considered in the electrical engineering field to overcome specific design problems caused by uncertainty [7,8]. The numerical evaluation of the effects of uncertainty is traditionally achieved by using the Monte Carlo method, which is widely accepted as an ‘‘exact method” for determining uncertainty [9]. The Monte Carlo method can give the entire probability density function (PDF) of any system variable through reiterations of the system simulation. Another method used for the evaluation of uncertainty is ‘‘true worst-case circuit tolerance” analysis, which gives only the results concerning the upper and lower statistical bounds of the circuit response to uncertainty [10]. The polynomial chaos approach to the evaluation of uncertainty also yields the full PDF of the system's variables using only one execution of the simulation. A method to automate the process of generating a circuit's polynomial chaos representation for use in simulation is presented in the paper.

It has been common practice in engineering to analyze systems based on deterministic mathematical models with precisely defined input data. However, since such ideal situations are rarely encountered in practice, the need to address uncertainties is now clearly recognized and there has been a growing interest in the application of probabilistic and other methods. Among the existing methods for uncertainty analysis, Ghanem and Spanos pioneered a polynomial chaos expan-

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sion method [11,12]. This method is based on the homogeneous chaos theory of Wiener [13], which uses a spectral expansion of random variables. The use of the term chaos in this context represents uncertainty and should not be confused with the chaos theory used frequently in theoretical physics. The homogeneous chaos expansion employs Hermite orthogonal polynomials in terms of Gaussian random variables. Cameron and Martin have proved that this expansion converges to any L_2 functional in random space in the L_2 sense [14]. Combined with Karhunen–Loeve decomposition [15] of the inputs, polynomial chaos results in computationally tractable algorithms for large engineering systems. Recently, a more general framework, called generalized polynomial chaos or Askey chaos, has been proposed [16]. This expansion technique utilizes more orthogonal polynomials from the Askey scheme than the original homogeneous chaos theory [17] and can represent general non-Gaussian processes. Applications to ODE, PDE, Navier–Stokes equations and flow-structure interactions have been reported and convergence has been demonstrated for model problems [18,19]. In recent years, polynomial chaos has been applied to measurement uncertainty [20,21], entropy multivariate analysis [22], control design [23–25], design of a two-planar manipulator [26] and polynomial chaos based observers for use in control theory [27]. Ref. [28] presents an overview of applications of polynomial chaos theory to electrical engineering. Among other possible applications, circuit simulation by means of polynomial chaos expansion is introduced. In that paper the problem is solved for a specific topology without providing a generalized algorithm for the solution of uncertain systems based on a SPICE-like netlist and information about the uncertainty of some parameters. The major contribution of this present work is in effect the formalization of the automatic solution process using resistive companion method [29]. In particular, a systematic algorithm to automatically define the conductance matrix of an uncertain system is presented. This theory yields the definition of a new approach to nodal analysis where the topology of a network is mapped to a multilayer topology where each layer represents one level of polynomial expansion. This approach helps to pave the way for a new generation of uncertainty based CAD tools.

2. Polynomial chaos

Wiener first introduced polynomial chaos theory in the form of homogeneous chaos expansion in 1938 [13]. This expansion uses a rescaled version of the Hermite polynomials, which correspond to a Gaussian or normal distribution when used in combination with Gaussian random variables. This expansion technique is based on independent random variables ξ_i, which are associated with an individual random event θ. Every uncertain parameter or variable in the examined system is represented by a random variable ξ_i. The total number of random variables is denoted by the symbol n_v. Each random variable is represented by a single variable polynomial expression in terms of ξ_i. While in principle each decomposition comprises an infinite number of terms, for practical purposes the polynomial order for each single variable polynomial is limited to a finite number of terms denoted by the symbol n_p. When the examined system contains multiple uncertain parameters, a single variable polynomial basis is no longer sufficient to represent the uncertainty of the system. The single variable polynomial contributions from each uncertain variable in the system, are combined into a multivariable polynomial that comprises all of the uncertainty in the system. Eq. (2.1) represents the order of the resulting multi-variable polynomial as a function of n_v and n_p

$$P = \left(\frac{(n_v + n_p)!}{n_v! n_p!}\right) - 1 \tag{2.1}$$

The variables n_p and P are both indexed starting at zero, representing the first polynomial order. All relevant variable definitions are listed in Table 1.

Every variable in the examined system must be expanded along the entire multi-variable polynomial basis. The individual polynomial terms of the basis will be denoted by the symbol Ψ. Eq. (2.2) represents a variable $Y(t)$ that is being expanded along the multi-variable polynomial basis.

$$Y(t) = \sum_{i=0}^{P} y_i(t)\Psi_i \tag{2.2}$$

The multi-variable polynomial basis mathematically describes a general second order random process $X(\theta)$ by (2.3) [16]

$$X(\theta) = a_0\Psi_0 + \sum_{i_1=1}^{\infty} a_{i_1}\Psi_1(\xi_{i_1}(\theta)) + \sum_{i_1=1}^{\infty}\sum_{i_2=1}^{i_1} a_{i_1 i_2}\Psi_2(\xi_{i_1}(\theta), \xi_{i_2}(\theta)) + \sum_{i_1=1}^{\infty}\sum_{i_2=1}^{i_1}\sum_{i_3=1}^{i_2} a_{i_1 i_2 i_3}\Psi_3(\xi_{i_1}(\theta), \xi_{i_2}(\theta), \xi_{i_3}(\theta)) \cdots \tag{2.3}$$

Notice that there is no limit on the number of terms in (2.3). This equations represents the case in which $P \to \infty$. The number of terms resulting from the summations in (2.3) ultimately equals P. For simplification purposes (2.3) can be rewritten as in (2.4)

$$X(\theta) = \sum_{i=0}^{P} \beta_i \Psi_i(\xi(\theta)) \tag{2.4}$$

The indexing of the variable Ψ in (2.4) and (2.2) is different than in (2.3) firstly because (2.4) represents a truncated decomposition, and secondly because in (2.4) each index i refers to a set of polynomial terms of the decomposition. Once the values of n_v and n_p have been chosen, the summations from (2.3) are expanded and Ψ variables are re-indexed according to their