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Intelligible factors for the yield curve☆

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ABSTRACT

We construct a factor model of the yield curve and specify time series processes for these factors, so that the innovations are mutually orthogonal. At the same time, the factors are such that they assume clear, intuitive interpretations. The resulting "intelligible factors" should prove useful for investment professionals to discuss expectations about yield curves and the implied dynamics. Moreover, they allow us to distinguish announced changes of the monetary policy stance versus monetary policy surprises, which we find to be rare. We identify two such events, namely September 11, 2001, and the Fed reaction to the sub-prime crisis of 2007.

1. Introduction

For people who are required to form expectations about complex objects, such as the term structure of interest rates, it is useful to reduce the dimensionality of the problem with the help of a few factors that describe the object. Ideally, these factors should have two properties. Firstly, they should have an *interpretation* that is easy to understand. For instance, when we talk about the yield curve, the short rate is an interpretable factor, and the slope or the long rate, too, are interpretable factors. We can discuss our expectations about these things and it is clear what it means. Secondly, the factors should be driven by processes that have innovations that are *mutually orthogonal*. It is difficult to form expectations about the long rate and the slope separately if the long rate and the slope are driven by shocks that are not independent from each other. In this paper, we say that factors that fulfill these two requirements of being interpretable and having mutually and serially orthogonal innovations, are *intelligible*.

The literature on the term structure of interest rates has produced a plethora of factor models, which can be divided into three lines. The first consists of models that start with a specification of some stochastic process for the short rate. They then derive the dynamics of the term structure by imposing arbitrage-freeness. Early representatives in this line of research are Pye (1966); Vasicek (1977), and Cox et al. (1981). These models have been extended to multi-factor models where yields depend on the factors in an affine fashion. Duffie and Kan (1996) and Dai and Singleton (2000) are two prominent examples in this domain. In these models, the factors are typically some abstract entities with no clear economic interpretation.

The second line consists of general equilibrium models that explain the term structure from first principles. The classic reference here is Cox et al. (1985). They explicitly derive an equilibrium term structure process as a function of the preferences of a representative utility maximizer and an assumed process of k factors that describe the production possibilities of the economy. These k factors have macroeconomic interpretations by construction. This model, therefore, is an early representative of the macro-finance literature that explores the relation between asset prices in general – or in this case, the term structure in particular – with observable or latent macroeconomic variables. The modern version of this literature (e.g. Ang and Piazzesi, 2003; Rudebusch and Wu, 2008; Mönch, 2008) is much less explicit about the general equilibrium underpinnings of the model. It takes up the idea that the short rate depends on a set of macroeconomic variables and imposes arbitrage-freeness to derive the implications for the relation between the term structure and these macroeconomic factors. By using exogenous variables to

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estimate the model, they achieve a better fit of the yield curve compared to the models with unspecified factors.

The third line of the literature consists of purely descriptive, empirical factor models. These models describe the term structure with the help of a few factors in order to facilitate communication about it. This literature is motivated more by the needs of the practitioner than by the interests of the economic theorist. A classic early reference to this type of literature is Litterman and Scheinkman (1991), who apply principal component analysis to yield curve data. This analysis generates orthogonal factors by construction. Typically, three factors suffice to describe the yield curve, and the corresponding loadings suggest an interpretation as 'level', 'slope', and 'curvature'. Another early contribution in this domain is Nelson and Siegel (1987). These contributions are completely static in the sense that they model the yield curve at a particular point in time. They do not, however, contain information about the dynamics of the yield curve and can therefore not be used for forecasting. Their main advantage over the theory-based models is their better fit.

Like Litterman and Scheinkman (1991), Bliss (1997) used the first three principal components to describe the term structure, but he was the first to analyze the dynamics of these factors using a vector autoregressive (VAR) specification. Diebold and Li (2006) have done the same with the Nelson–Siegel factors. They show that this combination of a set of easily interpretable factor loadings together with a simple stochastic process yields better forecasting performance than the dynamic theory-based models. The drawback is, of course, that it is not guaranteed that the model is arbitrage-free. Moreover, the factor innovations in the Diebold–Li model are not independent of each other. In other words, in the Nelson–Siegel model, one cannot discuss level innovations independently from slope innovations because these innovations are statistically not orthogonal. This jeopardizes interpretability. One can still use these models for forecasting, but the meaning of shocks to the individual factors is unclear.

In this paper, we address this issue. We construct a factor model in the tradition of the third line of research, but impose orthogonality of the factor innovations in addition to a clear interpretation of the loadings, i.e. we restrict our factors to be intelligible. More concretely, our loadings, which can be identified as 'long', 'short', and 'curvature' factors, follow a VAR process with mutually independent and serially uncorrelated innovations. In other words, we construct the factors in such a way that the innovations to our short factor, for instance, are uncorrelated with innovations to the other two factors.

In addition to the imposed intelligibility of the factors, our estimation reveals a dynamic structure which suggests a macroeconomic interpretation. We find that the curvature factor is a leading indicator of the short factor, while the long factor largely lives a life of its own. We will argue that this dynamic structure suggests that the curvature factor captures the intended (and communicated) medium term monetary policy stance, and the short factor captures surprise policy actions. We find that the curvature factor explains a much greater share of term structure movements than the other two factors. This is especially true for movements at the short end of the yield curve, which suggests that most monetary policy actions are well communicated by the Fed before they are actually executed. This interpretation relates our model to the macro-finance term structure literature. Indeed, our results confirm the finding of Mönch (2006) that curvature factor innovations are informative about the future evolution of the yield curve.

Two remarks about the dominant role of the curvature factor are in order here. According to principal component analysis (Litterman and Scheinkman, 1991), a factor with a more or less constant loading on all maturities (the level factor) is traditionally considered to be dominant. Why is this not the case in our model? First, our model does not feature a level factor. Our long factor is constrained to have zero loading at zero maturity. Second, in our model the long factor, and maybe also the short factor, are more important than the curvature factor in describing the *shape* of the term structure at any given day. Yet, curvature is the main driver of the *dynamics* of the term structure as captured by the VAR. In this sense, curvature is the dominant factor affecting the changes of the term structure, through its influence on the other two factors.

2. The model

Our basic approach consists of two steps: First, we formulate an interpretable factor model for the yield curve at one point in time. Second, we look at the common evolution of the factors over time and impose a dynamic structure which makes the factors intelligible.

2.1. A parsimonious model

We will use two related factor models for the yield curve. Let the raw factors $\theta = [\theta_1 \; \cdots \; \theta_F]'$ be an F-dimensional column vector. Let m denote time to maturity, and define the loadings of the factors $y(m) = [y_1(m) \; \cdots \; y_F(m)]$ as an F-dimensional function,

$$y_i(m) = \begin{cases} -\dfrac{1-\alpha_i^m}{m}, & \text{if } m > 0, \\ \log \alpha_i, & \text{if } m = 0, \end{cases} \tag{1}$$

with $0 < \alpha_i < 1$ for all $i \in \{2, \dots, F\}$, and $y_1(m) = 1$ for all m.[1] The loadings y are continuous monotonic concave functions of the maturity with negative values that converge to zero as $m \to \infty$. The α-coefficients gauge the curvature of the loadings. The yield curve at time t is described by

$$r_t = y\theta_t + \epsilon_t = \hat{r}_t + \epsilon_t, \tag{2}$$

where ϵ_t is the idiosyncratic and $\hat{r}_t$ the systematic component. We will only consider the model with $F = 3$,

$$\hat{r}_t = \theta_{1,t} + y_2\theta_{2,t} + y_3\theta_{3,t}. \tag{3}$$

Given yield curve data of a day, and given α, the factors of that day, θ_t, can be estimated by OLS. The best choice of α can be estimated with a nonlinear regression. Once we know the factors for each point in time, $\theta = [\theta_1 \; \cdots \; \theta_T]$, and building on the idea of Diebold and Li (2006) we can estimate a VAR model for θ.[2] The VAR model in the factors represents, together with the loadings y (parametrized by α), a complete description of the yield curve dynamics.[3] Therefore, it allows, among other things, to assess the dynamic reaction of the yield curve to factor innovations, which

[1] The functional form of these loadings is just one example of loadings that can be used for what we try to accomplish. We will later discuss some of the properties that these loadings must have, and especially, why the traditional Nelson and Siegel (1987) specification of the loadings, or the various modifications of it (Svensson, 1994; Bliss, 1997; De Pooter, 2007), are not suitable for our purpose.

[2] Diebold and Li (2006) focus primarily on univariate AR-models for each factor separately as they observe little cross-factor interaction.

[3] Our factor model can be viewed as a random coefficient time series model. Under this interpretation the factors are random coefficients which we restrict to follow a VAR process. The VAR structure for the factors is rather flexible and allows the introduction of additional features like a multivariate GARCH specification of the error terms along the lines of Bauwens et al. (1997). Although such an approach would allow to capture volatility clustering, we discard it because of the considerable cost in terms of additional parameters.