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A tale of two yield curves: Modeling the joint term structure of dollar and euro interest rates

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ABSTRACT

Modeling the joint term structure of interest rates in the United States and the European Union, the two largest economies in the world, is extremely important in international finance. In this article, we provide both theoretical and empirical analysis of multi-factor joint affine term structure models (ATSM) for dollar and euro interest rates. In particular, we provide a systematic classification of multi-factor joint ATSM similar to that of Dai and Singleton (2000). A principal component analysis of daily dollar and euro interest rates reveals four factors in the data. We estimate four-factor joint ATSM using the approximate maximum likelihood method of Aït-Sahalia (2002, forthcoming) and compare the in-sample and out-of-sample performances of these models using some of the latest nonparametric methods. We find that a new four-factor model with two common and two local factors captures the joint term structure dynamics in the US and the EU reasonably well.

1. Introduction

Financial markets have become increasingly globalized in recent years. Banks and institutional investors regularly borrow, lend and invest around the globe and take on huge fixed income positions in many different countries. These international fixed income positions create exposures not only to exchange rate risks, but also to interest rate risks in different countries.[1] The multiple sources of risks involved make risk management of these fixed income portfolios challenging. Therefore, characterizing the joint dynamics of exchange rates and interest rates in different countries is extremely important for banks and investors, who are interested in assessing and managing the risks in their portfolios, as well as for regulators, who are keen to understand the underlying risks for setting adequate bank capital requirements and monitoring systemic risks.

Our article contributes to the literature by providing one of the first studies of multi-factor affine models of the joint term structure of interest rates in the two largest economies in the world, the United States and the European Union. We focus on the US and the EU because of the dominance of their currencies in the international financial markets as well as the importance of dollar- and euro-denominated bond markets. Since the introduction of euro in 1999, the currency has been gaining dominance as one of the two major currencies in the world. Countries that have adopted the euro include Austria, Belgium, Finland, France, Germany, Greece, Italy, Ireland, Luxembourg, The Netherlands, Spain and Portugal. With more than 610 billion euros in circulation at the end of 2006, the euro has surpassed the US dollar in terms of cash value in circulation (Atkins, 2006). Among international issuers outside the eurozone, euros and dollars are the favorite currencies. Most international issuers choose to issue their bonds and notes in euros (43.5% of total volume) or in US dollars (40.5% of total volume), according to the European Central Bank (2004). Among different bond markets, the euro and dollar bond markets are the two most important ones. As of the end of 2003, eurozone domestic governments and corporations had an outstanding volume of US $ 5462 billion worth of euro-denominated bonds issued in their domestic countries (European Central Bank, 2004). This represents 22.3% of outstanding volume of domestic-issued debt among all

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[1] Previous literature largely focuses on whether to hedge the exchange rate risk in such portfolios. However, the portfolio values are driven by interest rates in both regions, in addition to the exchange rate risk.

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developed countries, and this size is second only to that of the United States.

Our analysis of the joint dollar–euro affine term structure models (ATSM) makes both theoretical and empirical contributions to the literature. Theoretically, our article systematically examines joint term structure models with four (both local and common) factors. In a domestic setting, Dai and Singleton (2000) provide a systematic classification of N-factor ATSM into $(N+1)$-subfamilies and derive the maximal model that nests existing models within each subfamily. However, international joint term structure models add another layer of complexity due to the presence of both local and common factors. We provide one of the first classifications of all four-factor joint ATSM within the maximally admissible classification scheme.

Empirically, we provide new evidence on the joint term structure using daily LIBOR and Euribor rates (and corresponding swap rates) and dollar/euro exchange rates between July 1999 and June 2003.[2] While Litterman and Scheinkman (1991) show that three factors are needed to capture the US term structure, we find that more than three factors are needed to capture the joint dollar–euro term structure dynamics focus on four-factor models in our estimation. One class of four-factor term structure models, with two common and two local factors, is particularly promising in terms of in-sample goodness of fit and out-of-sample density forecasting ability. The model that works the best is a four-factor affine model in which two local factors drive volatilities and two common factors follow Gaussian processes.

We conduct our empirical analysis in three steps. First, using principal component analysis, we examine the total number of factors and the numbers of common vs. local factors needed to capture the joint dollar–euro term structure. Domestic term structure models usually use up to three factors (e.g., Dai and Singleton, 2000). But in an international setting, how many factors should be included and how many of them should be common or local remain open questions. Most international articles use two or three factors with different combinations of local and common factors (e.g., Inci and Lu, 2004; Hodrick and Vassalou, 2002; Tang and Xia, 2006; Backus et al., 2001; Mosburger and Schneider, 2005). Motivated by empirical evidence from our principal component analysis, we go beyond the usual three-factor models to examine four-factor models.

Second, we estimate the models using the approximate maximum likelihood estimation, a powerful method developed recently by Aït-Sahalia (2002, forthcoming) and Aït-Sahalia and Kimmel (2002). The absence of a closed-form solution for transition density of affine models makes maximum likelihood estimation infeasible. Existing studies on international term structure models have resorted to other estimation methods, which could be either inaccurate in small samples or computationally burdensome. The approximate maximum likelihood estimation provides extremely fast and accurate estimations for affine models. We show that Aït-Sahalia's approximate maximum likelihood estimation techniques work very well in estimating international term structure models.

Third, we examine in-sample goodness of fit and out-of-sample density forecasting performances of all models using the nonparametric tests developed in Hong and Li (2005), Egorov et al. (2006), and Hong et al. (forthcoming). The difficulty in comparing different subfamilies of affine models, which generally are nonnested, has been recognized in the literature since Dai and Singleton (2000). Similarly, Tang and Xia (2006) note that "a suitable test in the current [non-nested] setting is…still not available in the literature". The nonparametric tests we consider overcome this difficulty by allowing direct comparisons of non-nested models. Moreover, these tests can reveal whether a given term structure model can satisfactorily capture the joint conditional densities of bond yields in both in-sample and out-of-sample settings. Therefore, unlike most existing studies that mainly focus on in-sample goodness of fit, we study the performances of joint ATSM in forecasting the joint conditional densities of bond yields, which are extremely important for managing international bond portfolios.

Though in this article we focus on the completely affine models of Dai and Singleton (2002), we can easily extend our analysis to the essentially affine models of Duffee (2002) and the more flexible risk premium specifications of Cheridito et al. (2006). Since we estimate our term structure models in conjunction with the exchange rate data, we can easily extend our analysis to examine the forward premium puzzle and exchange rate dynamics. While we focus on forecasting the joint conditional density of bond yields in this article, we can extend the approximate maximum likelihood to calculate and forecast the Value-at-Risk (VaR) of international bond portfolios.

Our article belongs to the recent fast-growing literature on two-country joint term structure models, which has mainly focused on the forward premium puzzle or the benefits of international diversification.[3] While these studies have made important contributions, our article is the first that examines the joint dollar–euro term structures. In addition, while each of these studies uses a different specification of the term structure model, we provide a systematic analysis of four-factor joint ATSM. While most existing studies mainly focus on in-sample fit, our article is the first that examines the out-of-sample density forecasting performances of joint ATSM.

This article is the closest in spirit to a concurrent working article, Mosburger and Schneider (2005). They investigate the performance of models driven by a mutual set of global state variables in explaining joint term structures in the US and the UK. They then discuss which mixture of Gaussian and square root processes is best suited for modeling international bond markets. They also derive necessary conditions for the correlation and volatility structure of mixture models to explain the forward premium puzzle and differently shaped yield curves. Our study is different due to our focus on the euro and dollar markets, the econometric methodology used for comparing non-nested models, and the use of four-factor joint term structure models.

The article is organized as follows. Section 2 describes the data and reports on the principal component analysis for dollar and euro term structures. Section 3 introduces our two-country affine term structure model and provides a systematic classification of four-factor affine models with both global and local factors. Section 4 discusses the econometric methods for estimating and testing joint dollar–euro term structure models. Section 5 presents empirical results, and Section 6 concludes the article.

2. Data description and principal component analysis

In this section, we provide necessary background information on the economies of the US and the EU, as well as the dollar/euro exchange rate. We also provide summary statistics on the dollar and euro term structure data. Finally, we provide principal component analysis on the number of factors that drive the joint term structure in the two economies.

[2] We use data from July 2003 to June 2006 to examine out-of-sample performance of our models.

[3] Backus et al. (2001), Inci and Lu (2004), Han and Hammond (2003), and Brennan and Xia (2006) look at the forward premium puzzle. Dewachter and Maes (2001) and Ahn (2004) examine the benefits of international diversification. Hodrick and Vassalou (2002) and Tang and Xia (2006) empirically examine term structures in multiple countries.